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Fourier Series and Ovals in Finite Geometry

A century ago, Hurwitz showed how Fourier series can be used to study geometric properties of curves in the Euclidean plane that bound convex sets. Ten years ago, Helmut Groemer wrote a book on the subject—*Geometric Applications of Fourier Series and Spherical Harmonics*. It turns out to be convenient to enlarge the scope to include all oriented smooth closed curves (in the plane) that have, for $0 \leq \phi \leq 2\pi$, exactly one point where its directed tangent makes an angle of ϕ with the positive x-axis. Such a curve has a parametrization $z(\phi)\colon \mathbb{R} \to \mathbb{C}$, where $z(\phi) = z(\phi + 2\pi)$ and the unit tangent vector is $e^{i\phi}$ for all $\phi \in \mathbb{R}$. These curves form a vector space over $\mathbb{R}$ that properly contains the boundaries of convex sets. Since many of the properties of such curves that one can investigate using Fourier series involve only algebra, much remains valid when $\mathbb{R}$ and $\mathbb{C}$ are replaced by some other field and its quadratic extension. In particular, I am interested in the geometry of the planes represented by the finite field $\mathrm{GF}(q^2)$—for x and y elements of the finite field $\mathrm{GF}(q)$ of order q, the point (x, y) corresponds to $z = x + iy$, where i is an element of $\mathrm{GF}(q^2)$ that is not in $\mathrm{GF}(q)$. In fact, there is a point to this generalization: the use of finite Fourier series sheds light on a 50-year-old problem of Beniamino Segre, who sought a way to classify the ovals of finite projective planes. An oval is defined to be a set of $q + 1$ points, no three on a line; it can be represented by a Fourier series that resembles that of its continuous kin.