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*A Sufficient Condition for Uniform Convergence of a New Class of Newton Interpolation Series in Two Complex Variables*

The name "Geddes series" refers to an extensive catalogue of new classes of series expansions for interpolating and approximating multivariate functions. The catalogue includes classes such as Geddes–Taylor series, Geddes–Fourier series, and a surprisingly large number of other classes. The general Geddes series scheme was invented by the presenter and named in honor of his thesis supervisor in 2003.

The simplest class of Geddes series is the Geddes–Newton series, which interpolates a function of two variables on the lines of a two-dimensional grid; when either variable is held fixed, a Geddes–Newton series reduces to a generalized Newton interpolation series in the free variable. Grids with infinitely many lines generate Geddes–Newton series with infinitely many terms. We conjecture that if the original function is analytic in two complex variables over a sufficiently large region, the resulting Geddes–Newton series converges uniformly to the original function on every sufficiently small compact set containing all the grid lines. We also conjecture that the rate of convergence is linear or superlinear.

This talk describes our progress to date in proving these conjectures. To that end, we will present a new contour integral remainder formula, new rigorous error estimates, and a new sufficient condition for uniform convergence at a linear or superliner rate. All that remains is to prove that this sufficient condition is always satisfied. We will also use Maple to present two applications which demonstrate the rapid convergence of Geddes–Newton series in practice:

(1) the fast and accurate evaluation of certain kinds of multiple integrals in four real dimensions, and

(2) the uniform approximation of special functions by elementary functions.

This talk presents joint research with Keith Geddes.