STEVEN VANDUFFEL, Katholieke Universiteit Leuven, and Fortis Central Risk Management, Belgium
Closed-Form Approximations for Constant Continuous Annuities

For a series of cash flows, the stochastically discounted or compounded value is often a key quantity of interest in finance and actuarial science. Unfortunately, even for the most realistic rate of return models, it may be too difficult to obtain analytic expressions for the risk measures involving this discounted sum. Some recent research has demonstrated that in the case where the return process follows a Brownian motion, the so-called comonotonic approximations usually provide excellent and robust estimates of risk measures associated with discounted sums of cash flows involving log-normal returns.

We will derive analytic approximations for risk measures in case one considers the continuous counterpart of a discounted sum of log-normal returns. Although one may consider the discrete sums as providing a more realistic situation than their continuous counterpart, considering the continuous setting leads to more tractable explicit formulas and may therefore provide further insight necessary to expand the theory and to exploit new ideas for later developments. Moreover, the closed-form approximations we derive in this continuous set-up can then be compared more effectively with some exact results, thereby facilitating a discussion about the accuracy of the approximations. Indeed, in the discrete setting, one must always compare approximations with results from simulation procedures, which always gives room for debate.

Our numerical comparisons reveal that the comonotonic "maximal variance" lower bound approximation provides an excellent fit for several risk measures associated with integrals involving log-normal returns. Similar results to those we derive for continuous annuities can also be obtained in the case of continuously compounding which therefore opens a roadmap for deriving closed-form approximations for the prices of Asian options. Future research will also focus on optimal portfolio slection problems.

This is joint work with Jan Dhaene and Emil Valdez.