GILLES PISIER, Texas A&M and Paris VI Similarity problems and amenability

In 1955, Kadison formulated the following conjecture: any bounded homomorphism  $u: A \to B(H)$ , from a  $C^*$ -algebra into the algebra B(H) of all bounded operators on a Hilbert space H, is similar to a \*-homomorphism, *i.e.*, there is an invertible operator  $\xi: H \to H$  such that  $x \to \xi u(x)\xi^{-1}$  satisfies  $\xi u(x^*)\xi^{-1} = (\xi u(x)\xi^{-1})^*$  for all x in A. This conjecture remains unproved, although many partial results are known. We will survey those as well as more recent results on the closely related notion of length of an operator algebra. In particular, we will explain why length equal to 2 characterizes amenable groups or nuclear  $C^*$ -algebras.