CRISTIAN ENACHE, Département de mathématiques et de statistique, Université Laval, Cité universitaire, Québec (Québec) G1K 7P4

Spatial decay bounds and continous dependence on the data for the solution of a semilinear heat equation in a long cylindrical region

In this work we study a semilinear heat equation in a long cylindrical region for which the far end and the lateral surface are held at zero temperature and a nonzero temperature is applied at the near end. In other words, the specific domain we consider is a finite cylinder $\Omega := D \times [0, L]$, where D is a bounded convex domain in the (x_1, x_2)-plane with smooth boundary $\partial D \in C^{2,\varepsilon}$, the generators of the cylinder are parallel to the x_3-axis and its length is L. The specific problem we consider is the following initial boundary value problem

$$\begin{cases} \Delta u - u_{,t} = -f(u), & \mathbf{x} \in \Omega,\ t \in (0, T), \\ u(\mathbf{x}, t) = 0, & \mathbf{x} \in \partial\Omega_L \cup \partial\Omega_{\text{lat}},\ t \in (0, T), \\ u(\mathbf{x}, t) = h(x_1, x_2, t), & \mathbf{x} \in \partial\Omega_0,\ t \in (0, T), \\ u(\mathbf{x}, 0) = 0, & \mathbf{x} \in \Omega. \end{cases} \tag{1.1}$$

where $\partial\Omega_0 := D \times \{0\}$, $\partial\Omega_L := D \times \{L\}$, $\partial\Omega_{\text{lat}} := \partial D \times (0, L)$. We also assume that $h(x_1, x_2, t)$ is a prescribed non-negative function with $h(x_1, x_2, 0) = 0$ and f is a non-negative function satisfying the following conditions

$$\lim_{s \to 0} \frac{f(s)}{s} \text{ exists, } f'(s) \le p(s) \text{ and } f''(s) \le q(s) \text{ for } s \ge 0, \tag{1.2}$$

where $p(s)$ and $q(s)$ are some non-decreasing function of s. We are interested on the spatial decay bounds for the solution of the semilinear heat equation (1.1) and on its continous dependence with respect to the data at the near end of the cylinder. Since the solution $u(\mathbf{x}, t)$ of the problem (1.1) can blow up at some point in space time, our aim is to derive sufficient conditions on the data which will guarantee that the solution remains bounded and moreover, under such conditions, we will obtain some explicit spatial decay bounds for the solution, its cross-sectional derivatives and its temporal derivative. We will also prove that the solution depends continously on the data $h(x_1, x_2, t)$ at the near end of the cylinder.