THOMAS HÉLIÈRE, Université Laval
Beyond Escobar's Conjecture

In 1999, Escobar conjectured that the first nonzero Steklov eigenvalue of a compact Riemannian manifold with nonnegative Ricci curvature and principal curvatures of the boundary bounded below by a constant $c > 0$ satisfies $\sigma_1 \geq c$. While this inequality is known to hold under the stronger assumption of nonnegative sectional curvature, the conjecture has recently been disproved. In this talk, I will present joint work with Alexandre Girouard in which we construct an explicit polynomial perturbation of the Euclidean metric on the three-dimensional ball that preserves positive Ricci curvature and strict boundary convexity, yet violates Escobar's conjectured inequality. This naturally raises a new question: what is the optimal lower bound for the first nonzero Steklov eigenvalue under Escobar's original geometric assumptions?