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*Regular Bisections of Nilpotent Subalgebras of* $\mathfrak{sl}(n)$

In this talk, we explore how nilpotent regular Lie subalgebras $\mathfrak{g}$ of the special linear algebra $\mathfrak{sl}(n)$ can be split into bisections, pairs of subalgebras of $\mathfrak{g}$ that equal $\mathfrak{g}$ as a vector space direct sum. These splittings help reveal internal structure of the $\mathfrak{g}$ and provide a concrete way to understand how its parts fit together, through a matrix representation of the bisections. There is a classical bijection between biclosed subsets of root systems and biclosed subsets of root systems. In this talk we will investigate a counting of bisections of the strict upper triangle algebra $\mathfrak{n}(n) \subseteq \mathfrak{sl}(n)$ which provides a new perspective on this bijection, and will discuss how this could lead to investigations using the Weyl group of a matrix representations of bijections, implicitly giving a group structure on all bisections any nilpotent regular subalgebra of $\mathfrak{sl}(n)$. There will be $n!$ bisections of $\mathfrak{n}(n)$ and this will be proven. The proof uses an induction argument that introduces the idea of extending algebras, and those extensions being complementary to each other. Only the bisections of some algebras can be represented as matrices, but all are relevant to the Weyl group as structure of bisections in general can be inherited by those that can be represented as matrices. Lastly, we present an algorithm, which we will call the falling terms algorithm, to count bisections of subalgebras of the strict upper triangle.