Nonlinear Waves and Dispersive PDEs: Recent Advances in Theory and Computations
Ondes non-linéaires et EDP dispersives: Avancées récentes théoriques et calculatoires
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Overhanging waves in the water wave equations with constant vorticity

In this talk, we present a computer-assisted methodology for proving the existence of periodic traveling gravity waves at the free surface of water, in a flow of constant vorticity over a flat bed. Using conformal mappings, the free-boundary problem is reformulated as a quasilinear pseudodifferential equation for a periodic function of a single variable. We then expand the solution in a Fourier series, reducing the problem to an infinite-dimensional system of equations for the Fourier coefficients.

To establish existence, we employ a Newton–Kantorovich type argument, proving the existence of a true solution in a neighborhood of a numerically computed approximation. The verification of the hypotheses of this fixed-point argument relies on a careful combination of analytical estimates and rigorous numerical computations.

This approach enables us to rigorously prove the existence of an overhanging wave, that is, wave profiles that are no longer graphs of functions. Moreover, it allows for the construction of branches of solutions parameterized by the relative mass flux, bifurcating from the trivial flat solution. In particular, we rigorously construct a branch that originates at the flat solution and extends to a wave with a self-intersecting profile, passing through overhanging configurations along the way.

This is a joint work with Susanna Haziot (Princeton).

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Nonlinear stability of periodic waves in the Benjamin-Ono equation

We construct the Lyapunov functional for the periodic waves of the Benjamin–Ono equation based on the conservation of momentum, energy, and a higher-order energy, which exists due to integrability. Positivity of the second variation of the Lyapunov functional for perturbations in Sobolev space $H^1(\mathbb{R})$ is established by utilizing the basis properties of eigenfunctions of the linearized Benjamin–Ono equation in $L^2(\mathbb{R})$. Coercivity of the Lyapunov functional for co-periodic and multiple-periodic perturbations yields the nonlinear orbital stability of the periodic waves in the periodic domain.

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