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Overhanging waves in the water wave equations with constant vorticity

In this talk, we present a computer-assisted methodology for proving the existence of periodic traveling gravity waves at the free surface of water, in a flow of constant vorticity over a flat bed. Using conformal mappings, the free-boundary problem is reformulated as a quasilinear pseudodifferential equation for a periodic function of a single variable. We then expand the solution in a Fourier series, reducing the problem to an infinite-dimensional system of equations for the Fourier coefficients.

To establish existence, we employ a Newton–Kantorovich type argument, proving the existence of a true solution in a neighborhood of a numerically computed approximation. The verification of the hypotheses of this fixed-point argument relies on a careful combination of analytical estimates and rigorous numerical computations.

This approach enables us to rigorously prove the existence of an overhanging wave, that is, wave profiles that are no longer graphs of functions. Moreover, it allows for the construction of branches of solutions parameterized by the relative mass flux, bifurcating from the trivial flat solution. In particular, we rigorously construct a branch that originates at the flat solution and extends to a wave with a self-intersecting profile, passing through overhanging configurations along the way.

This is a joint work with Susanna Haziot (Princeton).