Higher Category Theory Across Mathematics
La théorie des catégories supérieurs à travers les mathématiques
(Org: **Kristine Bauer** (University of Calgary) and/et **Martina Rovelli** (University of Ottawa))

JULIE BERGNER, University of Virginia
Classifying diagrams and equivariant operads

One description of the composition product for operads can be formulated elegantly in terms of the component spaces of the classifying diagram associated to a small category. The classifying diagram was originally developed as one way to regard a category as a complete Segal space, and can be thought of as a refinement of the ordinary nerve construction. In ongoing work with Calle, Chan, Osorno, and Sarazola, we want to develop a similar approach to a composition product for equivariant operads. For this purpose, we can use a similar construction using certain kinds of quotient maps in place of isomorphisms. We hope this construction will have a broader use in other equivariant constructions.

ELEFTHERIOS CHATZITHEODORIDIS, Virginia Commonwealth University
Models for rational $(\infty, 1)$-categories

An $(\infty, 1)$-category is a category enriched in spaces, possibly weakly. Our understanding of $(\infty, 1)$-categories has been advanced thanks to the development of various models for $(\infty, 1)$-categories, that is, mathematical objects that exhibit the structure of an $(\infty, 1)$-category. Two such models are complete Segal spaces, as introduced by Rezk, and Segal categories, as developed from the homotopical perspective by Bergner.

We introduce rational $(\infty, 1)$-categories, which are $(\infty, 1)$-categories enriched in spaces whose higher homotopy groups are rational vector spaces. We produce two models for rational $(\infty, 1)$-categories, rational complete Segal spaces and rational Segal categories. Our argument works for enrichment in general localizations of spaces, such as the v_n-periodic spaces of Heuts in unstable chromatic homotopy theory.

ROBIN COCKETT, University of Calgary
Affine schemes as a tangent restriction category: part I

In this series of two talks, we look at the partial map structure and tangent category structure of the category of commutative ri(n)gs opposite, and how these structures interact.

GEOFF CRUTTWELL, Mount Allison University
Affine schemes as a tangent restriction category (part II)

In this series of two talks, we look at the partial map structure and tangent category structure of the category of commutative ri(n)gs opposite, and how these structures interact.

PEDRO BRUNIALTI LIMA DE ANDRADE, University of Virginia
Involutions on (∞, n)-categories of Bordisms and Reflectopm TQFTs

Formally, a topological quantum field theory (TQFT) is a symmetric monoidal functor from a category of bordisms into an appropriate target category. Depending on the choice of bordism category, the theory may carry extra structure compatible with the monoidal structure — namely, an involution. By involution, we mean a $\mathbb{Z}/2$ action on the bordism category which is compatible with the monoidal structure. In this talk, I will outline how to get these actions on the (∞, n)-categories of bordisms defined by Calaque and Scheimbauer, and I will define reflection TQFTs, in the sense of Freed and Hopkins, in this setting. Furthermore, I will exemplify what kind of obstructions a reflection structure imposes on the point values of a theory.

ARGHAN DUTTA, University of Ottawa
Symmetric monoidal (∞, n)-categories

Symmetric monoidal (∞, n)-categories have been considered in a variety of forms and from several perspectives, including Lurie's commutative algebra objects and, for $(\infty, 1)$-categories, coCartesian fibrations over finite pointed sets satisfying a Segal condition; Gepner and Haugseng's iterated enrichment; and the special Γ-objects of Schommer-Pries, Calaque–Scheimbauer, and Johnson-Freyd.

In this talk, I will introduce a definition in terms of left double (∞, n)-fibrations, building on the framework for double higher categories developed by Moser, Rasekh, and Rovelli. Working internally in a suitable model category presenting (∞, n)-categories, we express the symmetric monoidal Segal condition through spans associated with the inert maps of finite pointed sets. I will then focus on the quasi-categorical case, where we prove a Quillen equivalence with Lurie's presentation via coCartesian fibrations; in the strict one-dimensional case, our approach recovers the ordinary symmetric monoidal categories. Our comparison uses strings of coCartesian morphisms and a staircase construction to pass between the two descriptions. I will explain the ideas behind these constructions and how they relate the two Segal conditions for symmetric monoidality.

MARTIN FRANKLAND, University of Regina
Near-ringoids of operations

When we consider unary operations acting on homotopy groups or cohomology groups, there's an issue with the algebra of operations: it's not an algebra. (a) Operations between different degrees give a "many objects" structure; that's the "oid" part. (b) The presence of non-additive operations means that composition is only linear in one variable rather than bilinear; that's the "near" part. We will discuss some of the algebraic structure of near-ringoids, as well as examples from homotopy theory. Based on joint work with Manak Singh.

ADINA GOLDBERG, Fields Institute
A double category of quantum sets, functions and relations

We introduce the double category $\mathbb{Q}$**Rel** of finite quantum sets, functions, and binary relations, building on the work of Weaver, Kornell, and Musto-Reutter-Verdon. It is a candidate for a †-compact double category in the sense that there is a † structure on squares that is compatible with the duals structure. In analogy to the classical case of the double category $\mathbb{R}$**el** of functions and relations, the 2-categories **QSet** of finite quantum sets and quantum functions, and **qRel** of finite quantum sets and binary relations arise as the horizontal and vertical 2-categories of $\mathbb{Q}$**Rel**.

Time permitting, we illustrate how $\mathbb{Q}$**Rel** models both new and existing quantizations of some constructions from classical graph theory and quantum information theory.

STEFANO LUNEIA, University of Ottawa
Morita $(\infty, 2)$-categories and beyond

One difficulty in constructing an (∞, n)-category is showing that the different ways of composing morphisms fit together coherently. Bimodules give a familiar example. Viewed as arrows between rings, they compose by tensor product over the middle ring. This operation is associative up to canonical isomorphism, and those isomorphisms must be compatible when we compose longer strings of bimodules. Complicial sets express these compatibilities in terms of simplices and their faces. I will explain how my coauthors and I use this idea to construct the Morita $(\infty, 2)$-category of a monoidal category admitting a calculus of balanced tensor products.

I will then discuss work in progress on extending this construction to obtain an $(\infty, 3)$-category from a symmetric monoidal category. Time permitting, I will close with a proposal for an (∞, ∞)-category of manifolds and cobordisms, fully extended down to points and admitting iterated cobordisms in arbitrarily high dimensions.

DORETTE PRONK, Dalhousie University
A category representing Bredon cohomology for a G-space

Bredon-Illman cohomology with local coefficients for spaces with an action of a compact Lie group was defined by Mukherjee and Mukherjee in 1996. However, the definition is rather involved, which makes it difficult to prove general results about it. More recently, Farsi, Scull and Watts have shown that this cohomology is Morita invariant, making it a suitable homotopy invariant for orbifolds. They also slightly reformulated the definition. I will use this reformulated description to construct a category $\Delta_G(X)$ for each G-space which represents the Bredon cohomology with both constant and local coefficients for this space in the sense that the cohomology of the nerve with coefficient functor factoring through the discrete tom Dieck fundamental groupoid is the Bredon-Illman cohomology of the G-space with these coefficients. I will discuss some properties of this category and derive results about Bredon-Illman cohomology that can now easily be obtained. This is joint work with Laura Scull from Fort Lewis College.

CARLO MARIA SCANDOLO, University of Calgary
A general type-theoretic approach to higher-order quantum resources

Quantum resources exist in a hierarchy of multiple levels. At order zero, quantum states are transformed by linear maps (channels, or gates) in order to perform computations or simulate other states. At order one, gates and channels are transformed by linear maps (superchannels) in order to simulate other gates. To develop a full hierarchy of quantum resources, beyond those first two orders, and to account for the fact that quantum protocols can interconvert resources of different orders, we need a theoretical framework that addresses all orders in a uniform manner. We introduce a framework based on a system of types, which label the different kinds of objects that are present at different orders. Subsequently, we equip the framework with a parallel product operation that modifies and generalizes the tensor product so as to be operationally meaningful for maps of distinct and arbitrary orders.

PRIYAA VARSHINEE SRINIVASAN, University of Calgary
Drazin adjoints

Drazin inverses, a type of generalized inverses, concerns endomaps which are invertible after a certain number of iterations. These structures have been studied extensively in ring and semi-group theory, and most recently in category theory.

Weakening the rules of Drazin inverse produces a certain sort of weak adjunction, which we call Drazin adjunction — a situation when an object in a (symmetric) monoidal category is dual to another "in the presence of a context".

In this talk, we will present Drazin inverses and their generalization called Drazin adjoints and contrast their algebraic properties. This is joint work with Robin Cockett.

This talk is aimed to be category theory beginner friendly.

KIMBALL STRONG, Cornell University/Western University
Higher Categorical Perspectives on Homotopy Groups of Spheres

One of the loveliest mathematical descriptions I've heard is "the homotopy groups of spheres sort of sound like a lot of people screaming." This, of course, prompts the question: what's causing all that screaming? This talk will take the viewpoint that the homotopy groups of spheres arise as shadows of coherence axioms in weak higher groupoids. We will illustrate this with some low-dimensional examples and general theorems, and then sketch some future goals.

DANIEL TEXEIRA, Dalhousie University
Higher categories with adjoints via fibrations

We compare the Gepner-Heine proposal of (∞, ∞)-categories with adjoints via bicartesian fibrations to the usual definition,

where every k-morphism is required to have both adjoints. We also apply this approach to n-categories and categorical spectra with adjoints, and show that the total category of a cocartesian fibration has adjoints whenever its base and fibres do.

DAVID WHITE, Queen's University
A Recognition Principle for Diagram Categories

When is an enriched model category M Quillen equivalent to a category of enriched presheaves with the projective model structure? In joint work with Boris Chorny, we show that this occurs if and only if M has a set of "homotopy atoms," i.e., objects whose derived mapping functors jointly detect weak equivalences and preserve certain weighted homotopy colimits. Our result extends Marta Bunge's recognition theorem and generalizes the orbits of Dwyer and Kan. As an application to functor calculus, I will explain how reduced n-excisive functors from finite pointed simplicial sets to spectra can be modeled by a diagram category.

INNA ZAKHAREVICH, Cornell University
The K-theory of abelian monoid sets

The K-theory of sets equipped with a group action is well understood. Given a finite abelian group A, the K-theory theory of A-sets splits as a wedge of classifying spaces if groups A/H, for $H \leq A$. When A is ab abelian monoid, however, the case becomes much more complicated, because the orbits of an element of a monoid set are neither disjoint nor semisimple. This talk will explain the resolutions to these difficulties and show that the K-theory of abelian monoid sets also splits as a wedge of classifying spaces.