Functional and Geometric Inequalities
Inégalités fonctionnelles et géométriques
(Org: **Ryan Gibara** and/et **Scott Rodney** (Cape Breton University))

NICHOLAS BIGLEY, McMaster University
On Regularity of Landau-de Gennes Minimizers Around Nearly Touching Spherical Colloids

We study the Landau-de Gennes energy functional,

$$\mathcal{E}_{\xi,\eta}(Q) = \int_\Omega \frac{1}{2}|\nabla Q|^2 + \frac{1}{\xi^2} f(Q) + \frac{1}{\eta^2} g(Q)\, dx,$$

outside two spherical colloids separated by a distance $d > 0$ in the large particle limit while imposing normal boundary conditions and prove $L^\infty(\Omega)$ and Lipschitz estimates for minimizers of this system where the two colloids approach each other in the limit $d \to 0$.

ALMAZ BUTAEV, University of the Fraser Valley
Whitney extension theorem and limits at infinity

We will discuss the classic Whitney extension theorem and some of its variations preserving the behaviour at infinity. This is joint work with Nathan Strong.

ŞEYMA CETIN, Istanbul Gelisim University
On $H = W$ in Banach Function Spaces

In this talk, I will present some of our recent work—conducted in collaboration with D. Cruz-Uribe (UA, USA) and S. Rodney (CBU, Canada)—on what is known in the literature as the $H = W$ problem. We consider a Banach function space $X(\Omega)$ and its corresponding Sobolev space $W^1_X(\Omega)$.

Our central question is when is the space $W^1_X(\Omega)$ equal to the space $H^1_X(\Omega)$. Here, $H^1_X(\Omega)$ is defined as the closure of the set $\mathrm{Lip}(\Omega) \cap W^1_X(\Omega)$ with respect to the norm $\|f\|_{W^1_X(\Omega)} = \|f\|_{X(\Omega)} + \|\nabla f\|_{X(\Omega)}$.

I will discuss the hypotheses we impose on $X(\Omega)$ that allow this equality to hold. Specifically, we assume the Fatou property, the absolute continuity of the norm, and a strong Muckenhoupt condition. I will state our main result under these assumptions and briefly explain why they are essential to the proof.

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MOHAMED EL AOUMARI, Concordia University
Wavelet and tent space characterizations of $h^p(\mathbb{R}^n)$ and maximal admissible functions for Goldberg splittings

We present a two-part study of the local Hardy spaces $h^p(\mathbb{R}^n)$, $0 < p \leq 1$. The first part gives wavelet and tent space characterizations of h^p through a sequence space t_p on pairs of coefficients, a father layer on the unit lattice and a mother layer on the dyadic cubes. An exact isometric compression maps the tent space T_p into t_p. The characterization comes with an atomic decomposition and with exact L^2 convergence of the wavelet expansion. The second part asks which functions can serve in Goldberg-type splittings of h^p. We determine the maximal admissible class along three axes, regularity, decay and cancellation, and show that the endpoints are sharp. The proof rests on a molecular theory at critical decay, where a logarithmic approximate-cancellation condition appears at the critical moment order, and we prove that its rate cannot be improved. When $p = 1$ no regularity is required.

JOSH FLYNN, MIT
Some Braided Functional Inequalities

Several sharp inequalities comparing weighted L^p norms of a function $f : \mathbb{R}^n \to \mathbb{C}$ and its derivatives naturally extend with the same sharp constant to maps $u : \mathbb{R}^n \to A_Q(\mathbb{C})$, where $A_Q(\mathbb{C})$ is the space of unordered Q-tuples of complex numbers. In this talk we explore how topological constraints on the graphs of such maps can affect sharp constants and extremizers. We focus on Hardy and, more generally, Caffarelli-Kohn-Nirenberg inequalities. Broadly speaking, we explore the effects of generalizing known functional inequalities to inequalities for multiply-valued functions whose sheets suitably braid. AI disclaimer: these results were developed with the assistance of AI tools.

MANISHA GARG, University of Illinois Urbana-Champaign

DAMIR KINZEBULATOV, Université Laval
Strong solutions of SDEs with critical discontinuities in diffusion coefficients

We prove strong existence for Itô SDEs with diffusion coefficients that can introduce strong attraction to a submanifold. We extend and strengthen the Roeckner-Zhao approach, which uses Malliavin calculus to establish compactness of the approximating solutions in Wiener-Sobolev space. At least when the diffusion coefficients are sufficiently regular in time, this provides an alternative to Krylov's recent proof of strong existence via analysis of the Itô-Duhamel series. Joint work with Sallah Eddine Boutiah (Laval).

ILYA KRISHTAL, Northern Illinois University
Total Positivity and Near-Optimal Sampling Inequalities

A fundamental question in dynamical sampling is how many observations are needed for the stable recovery of a vector from its evolution under a linear operator. We investigate this problem for positive operators, highlighting the relation between the number of necessary observations and the spectrum of the operator.

Our main tool is a uniform interpolation theorem for translates of certain totally positive functions. We show that, given any uniformly separated sequence of centers, interpolation nodes can be chosen nearby so that the corresponding interpolation operator is boundedly invertible, with bounds depending only on the kernel and the minimum separation. Applying this result to operator orbits, we construct weighted Riesz bases with sampling counts attaining the natural spectral lower bound up to a bounded additive error.

The talk is based on joint work with Jian Wu.

LIANGBING LUO, Queen's University
Discrete Approximation on Metric Measure Spaces of Controlled Geometry via Graphs: Dirichlet Forms and Harmonic Functions

Based on the approximation of a metric measure space via discrete graphs, we construct a Dirichlet form. This Dirichlet form is comparable to the upper gradient energy and can be realized as a Gamma-limit of a sequence of induced bilinear forms projected from the discrete energy form on the approximating graphs. Moreover, we approximate harmonic functions on a bounded domain with a prescribed Newton-Sobolev boundary data. Based on joint work with A. Butaev and N. Shanmugalingam.

JAVAD MASHREGHI, Université Laval

MARIA NTEKOUME, Concordia University
Homogenization results for nonlinear Schrödinger equations

The cubic nonlinear Schrödinger equation (NLS) arises as a model for the propagation of intense continuous wave laser beams in a homogeneous medium. In practice, it has also been successful in modeling optical experiments in inhomogeneous settings. This suggests the occurrence of homogenization, that is, in a large scale limit, solutions to the inhomogeneous equation converge to the solution of a homogeneous NLS. We will review some recent homogenization results for physically motivated examples. While materials with a periodic structure are the most natural setting to consider this problem, our examples include a model where strong defects are sprinkled randomly across the medium. Part of this talk is based on joint work with B. Harrop-Griffiths.

CINTIA PACCHIANO, Universidad Nacional Autónoma de México
Beyond Classical Poincaré Inequalities: Double-Phase Problems in Metric Spaces

In this talk, I will discuss different Poincaré and Sobolev-Poincaré inequalities that arise in the study of double-phase problems in metric measure spaces. These problems involve functionals with nonstandard growth, where the energy depends on two different powers of the gradient and a spatially varying coefficient. I will focus on how the growth exponents, the regularity of the coefficient, and the properties of the underlying space affect the inequalities we can establish. I will also explain how these estimates are used to obtain higher integrability and regularity results for quasiminimizers.

The talk is based on joint works with Juha Kinnunen (Aalto University) and Antonella Nastasi (University of Palermo).

BLANCA RADILLO-MURGUIA, University of British Columbia
A Covering Lemma in the Theory of Differentiation of Integrals

We will explore the interplay between geometric maximal functions and covering lemmas. Motivated by the work of A. Córdoba and R. Fefferman, we will discuss a necessary and sufficient covering condition on a basis that yields a weak-type estimate for the associated maximal operator. This is joint work with Paul Hagelstein and Fedor Nazarov.

SCOTT RODNEY, Cape Breton University
Degenerate Sobolev and Poincare Inequalities via Extrapolation

In this talk I will describe recent results with F. Dal (YTU IST, Turkiye) and D. Cruz-Uribe (UAl). For a possibly degenerate symmetric non-negative semidefinite matrix valued function Q(x) on a bounded domain Ω of $\mathbb{R}^n$ that satisfies varying versions of the ellipticity condition

$$w(x)|\xi|^p \leq |\sqrt{Q(x)}\xi|^p \leq v(x)|\xi|^p$$

for a $p \in (1, \infty)$ and weights w, v, we show that a Sobolev inequality of the form

$$\|f\|_{L^{p\tau}(v,\Omega)} \leq C\|w^{-1}\|_{L^s(\Omega)}\|v\|_{L^r(\Omega)}\|\sqrt{Q}\nabla f\|_{L^p(\Omega)}$$

holds for constants $C, 1 \leq r, s \leq \infty$ and $\tau \geq 1$ independent of $f \in Lip_c(\Omega)$. Our techniques do not require any doubling conditions on the weights nor do they ask for Mukenhoupt A_p conditions. Our techniques rely on ideas from the theory of Rubio De Francia extrapolation in harmonic analysis.

OPEN PROBLEM SESSION,

ALINA STANCU, Concordia University
Surface Area Minimization with Prescribed Antipodal Chords

We study the problem of minimizing the surface area of star-shaped bodies with a prescribed antipodal chord function. Unlike the planar case, the centrally symmetric body may not minimize surface area in higher dimensions. We will present sufficient conditions for symmetric minimization and some examples of symmetry breaking.

CODY STOCKDALE, Clemson University
A dimension-free weak-type $(1,1)$ bound for the vector Riesz transform on $\mathbb{R}^n$

We show that the best constant in the weak-type $(1,1)$ bound for the vector Riesz transform on $\mathbb{R}^n$ is at most 2, independent of the dimension n. The proof relies on a new decomposition of the input data involving an obstacle problem for the fractional Laplacian and an associated Lewy-Stampacchia type estimate on an unbounded domain. This settles a problem posed by E. M. Stein at the 1986 International Congress of Mathematicians.

IGNACIO URIARTE-TUERO, University of Toronto
Recent progress related to the Krzyż conjecture

Let f be an analytic function from the unit disk into the unit disk, which is non-vanishing. Expanding in Taylor series about the origin, $f(z) = \sum_{n=0}^{\infty} a_n z^n$. Krzyż conjectured in the 1960s that for any fixed $n \geq 1$, the supremum of $|a_n|$ in this class of functions is precisely $\frac{2}{e}$ (independent of n!!) and that the supremum would only be attained for a certain function (up to normalizations). The conjecture has been proved for $1 \leq n \leq 7$ ($n = 6, 7$ have been recent proofs, AI assisted), but there have been several other recent advances. I will explain some of these advances: some are by other authors, some are joint work with Eiframidis, Llinares, MacDonald and Vukotic, some joint with Eiframidis, Llinares, Martin, Sawyer and Vukotic, and some are solo work by my student MacDonald.

MANASA VEMPATI, Louisiana State University

JULIAN WEIGT, University of Oulu
Sharp regularity of maximal functions

The classical Hardy-Littlewood maximal function theorem states that the Hardy-Littlewood maximal operator is a bounded operator on $L^p(\mathbb{R}^d)$ if and only if $1 < p \leq \infty$. The main ingredient to the proof is the Vitali covering lemma, which roughly states that any bounded set of balls has a pairwise disjoint subset which represents the volume of the original set well.

In 1997 Juha Kinnunen proved the corresponding result for the gradient of the maximal function, i.e. that the $L^p(\mathbb{R}^d)$-norm of the gradient of the maximal function is controlled by the $L^p(\mathbb{R}^d)$-norm of the gradient of the function if $1 < p \leq \infty$. However, he provides no counterexample in the endpoint $p = 1$, and so in 2004 Hajłasz and Onninen formally posed the question if the endpoint gradient bound also holds.

We present current research on the problem, emphasizing the one dimensional case $d = 1$.

ALEXIA YAVICOLI, University of British Columbia

CHENGJUN YUE, Université Laval
Kinetic SDEs and kinetic Fokker-Planck equations with critical Morrey-type drifts, and local blow-ups

In this talk, I will present a detailed weak solution theory for kinetic SDEs with singular drifts in a broad class that, in particular, can produce local blow-ups due to strong position-velocity alignment. The condition on the drift is given e.g. by essentially the largest possible critical Morrey class. For our proofs, we develop a new approach to the Sobolev theory of kinetic forward (Fokker-Planck) and backward equations based on a nonlocal decomposition of the Duhamel series and kinetic Adams-type estimates. This is joint work with Jean-François Jabir and Damir Kinzebulatov.

ZHICHUN ZHAI, MacEwan University

Sobolev Extension and Trace-Type Inequalities via Fractional Dissipative Equations

Analytic inequalities, particularly Sobolev inequalities, play a fundamental role in the study of partial differential equations. In this talk, we will demonstrate how fractional dissipative equations give rise to Sobolev extension and trace-type inequalities. We regard the solutions of fractional dissipative equations as functionals of their initial data. Using capacity methods, we characterize the extension from the initial data to the corresponding solutions. Conversely, by viewing the initial data as the trace of the solution, we establish corresponding Sobolev trace-type inequalities.