Early-Career Researchers in Graph Theory
Chercheurs en début de carrière en théorie des graphes
(Org: **Kathie Cameron** (Wilfrid Laurier University), **Cicely Henderson** (University of Waterloo) and/et **Taite LaGrange** (University of Waterloo))

ALEXANDER CLOW, Simon Fraser University
Ultimately, How Independent is the 5-Wheel?

The ultimate independence ratio of a graph G is defined as $\mathcal{I}(G) = \lim_{k\to\infty} \frac{\alpha(G^{\Box k})}{|V(G)|^k}$, where $\alpha(G^{\Box k})$ is the independence number of the Cartesian product of k copies of G. For all graphs G, Hahn, Hell, and Poljak (1995) proved that $\frac{1}{\chi(G)} \leq \mathcal{I}(G) \leq \frac{1}{\omega(G)}$ where $\chi(G)$ is the chromatic number, and $\omega(G)$ is the clique number of G. Motivated by this, the same authors conjectured that for all odd wheels $\mathcal{I}(W_{2t+1}) = \frac{1}{4}$. By combine counting arguments, novel recursive bounds on $\alpha(G^{\Box k})$ which yield generalized inequalities for $\mathcal{I}(G)$, and computer-assisted calculation in the W_5 case we provide the first progress on this conjecture in three decades.

This is joint work with Hitesh Kumar and Shivaramakrishna Pragada.

JULIEN CODSI, Princeton University
Between the Grid Minor Theorem and the Pathwidth Theorem

One way to interpret the grid minor theorem is the following: forbidding a planar graph that is minor-universal for all planar graphs up to a given size forces the treewidth to be bounded. Similarly, the pathwidth theorem tells us that forbidding a tree that is minor-universal for all trees up to a given size forces the pathwidth to be bounded.

From this perspective, it is natural to ask what happens between these two. What can we say if we forbid a minor-universal outerplanar graph? What about a minor-universal series-parallel graph? In this talk, I will present some partial answers to these questions, filling part of the gap between trees and planar graphs

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XINYUE FAN, University of Waterloo
Minimum Degree and Treewidth: Algorithm, Complexity and Structure

How densely can a network be connected while still maintaining a certain level of simplicity in its structure? And how difficult is it to add connections to a network so that every vertex has enough connections, without making the overall structure too complicated?

In this talk, we explore these questions through the mathematical study of graphs, which provide a way to represent networks of interconnected objects. We focus on two competing requirements: ensuring that every vertex has a minimum number of connections, while keeping the network structurally simple. We measure this simplicity using a concept called treewidth, which, roughly speaking, captures how closely a network resembles a tree.

We first determine exactly when it is possible to construct a connected network that meets these requirements with the smallest possible number of edges. Time permitting, we will investigate a related problem: given an existing network, what is the minimum number of new connections needed to ensure that every vertex has sufficiently many connections while keeping its structure simple?

CICELY HENDERSON, University of Waterloo
Hypergraph Hajnal-Szemerédi

A seminal result of Hajnal and Szemerédi from 1970 asserts that every graph on n vertices with $q \mid n$ and minimum degree at least $\left(1-\frac{1}{q}\right)n$ admits a K_q-factor (a collection of vertex-disjoint copies of K_q spanning all vertices). There has been sustained

interest in extending this result to hypergraph K_q^r-factors. For general q and r, Lu and Székely in 2007 showed that every large enough, r-graph G with $q \mid v(G)$ and $\delta_{r-1}(G) \geq \left(1 - \frac{c}{q^{r-1}}\right) v(G)$ admits a K_q^r-factor (where c is a constant depending on r but not q), and there have been various subsequent improvements to the constant (but not the order) of this bound over the ensuing decades. Meanwhile, results from codegree Turán theory show that $\delta_{r-1}(G) \geq \left(1 - \frac{c \cdot \log q}{q^{r-1}}\right) v(G)$ is necessary. Surprisingly, our main result shows that the order of this upper bound is not correct and, in fact, the codegree K_q^r-factor threshold is much closer to this codegree Turán theory bound. In particular, we show that $\delta_{r-1}(G) \geq \left(1 - c \cdot \frac{\log^{1-1/(r-1)} q}{q^{r-1}}\right) v(G)$ suffices for the existence of a K_q^r-factor. Joint work with Michelle Delcourt, Thomas Lesgourgues, and Luke Postle.

LORA HREISH, University of Toronto
Cycles in the random d-process

The random d-process is a random time-evolving process introduced by Erdős in the 1980's. The process starts with n isolated vertices and at each step picks a pair of vertices uniformly at random. If both vertices have degree $\leq d-1$ and were not picked together before, an edge is added between them. The process runs until no more edges can be added.

Ruciński and Wormald proved in their paper *Random Graph Processes with Degree Restrictions* that whp the final result of the random d-process is d-regular (given dn is even). In 1997, the same authors proved in their paper *Random Graph Processes with Maximum Degree* 2 that the number of short cycles in the random 2-process is asymptotically Poisson with constant mean. Their proof, however, only applies to the special case $d = 2$ where the components of the evolving graph are either paths or cycles. We generalize this result to all $d \geq 2$. We do this by exploiting the fact that the random d-process is a time-evolving process and by using switching arguments on an easier version of the process.

SEAN KIM, Simon Fraser University
Cycle Lengths in Graphs of Large Chromatic Number

In 2016, Brewster, McGuinness, Moore, and Noel conjectured that for $k \geq 3$, a graph with chromatic number greater than k must have at least as many cycles of length $0 \bmod k$ as the complete graph on $k+1$ vertices. We proved that the conjecture is true when $k = 3$, and that K_4 was the unique minimizer over all 4-critical graphs. This talk will explore three topics: our proof for the $k = 3$ case, a recent proof of the entire conjecture, and further directions that can and have been explored regarding this topic.

TAITE LAGRANGE, University of Waterloo
Graph Products and Variants of Twin-width

Twin-width is a graph and matrix parameter introduced in 2021 by Bonnet, Kim, Thomassé, and Watrigant as essentially a measure of the 'error' between vertex neighbourhoods over a series of vertex contractions. While standard twin-width measures how these errors interact via vertex degrees, we may want to modify what properties of these errors we count. Total twin-width is a variant of twin-width that counts the total number of errors that occur at any given point in the contraction sequence. Pettersson & Sylvester previously showed that the tensor product of any two graphs G_1 and G_2 has twin-width at most $\max\{tww(G_1) \cdot \Delta(G_2) + \Delta(G_2), tww(G_2)\} + \Delta(G_2)$ but nothing was known of how the tensor product affects twin-width variants. In this talk, we show that for a graph G, the total twin-width of the tensor product of G with K_2 is at least the total twin-width of G. We also show that there exists an infinite family of graphs $\mathcal{C}$ with $ttww(G) = 0$ for any graph $G \in \mathcal{C}$ and $ttww(G \times K_2) \geq 2^c$ for all constants c. This is joint work with Therese Biedl and Sophie Spirkl.

KYNE SANTOS, Toronto Metropolitan University
Structural Change and Random Graph Models in Global Oil Trade Networks

Using UN Comtrade data on the bilateral trade of crude oil from 1988 to 2025, we examined temporal changes in network centrality measures and modularity. We also compared the oil trade network with several random graph models using 3- and

4-node subgraph profiles and machine learning classification. The oil trade network was consistently classified as a Chung-Lu configuration graph, suggesting that a model based on the degree distribution better matched its subgraph profiles than the other models considered.

AGNES TOTSCHNIG, Massachusetts Institute of Technology