Discrete-Time Processes on Graphs
Processus en temps discret sur les graphes
(Org: **Nancy E. Clarke** (Acadia University), **Alexander Clow** (Simon Fraser University) and/et **Erin Meger** (Queens University))

CAELAN ATAMANCHUK, McGill University
Burning and covering random trees

Graph burning is a discrete-time process on a graph, where vertices are either burning or not. In each round, a burning vertex causes all of its neighbours to burn, and one new fire is started at a chosen vertex. The process runs until all vertices are burning. The main quantity of interest is the burning number, which is the minimum number of rounds required to burn all vertices of the graph. The problem of upper bounding the burning number of various large families of graphs has been a popular topic in recent years.

In this talk, we will discuss bounds on the burning numbers of random trees (specifically size-conditioned Bienaymé trees). The bounds are a consequence of a law of large numbers for the covering numbers of random trees across various scales, which will be the main focus.

The talk is based on joint work with Simon Briend, Serte Donderwinkel, and Elaine Herrera.

ANDREA BURGESS, University of New Brunswick
Deduction with k moves

Deduction is a graph process in which searchers seek to capture an invisible evader on the vertices of a graph G in a series of stages. Vertices that have ever been occupied by a cop are considered protected. In each stage, searchers on a vertex v move to neighbouring vertices if the number of cops on v is at least the number of unprotected neighbours of v. However, each searcher may move a fixed number of times. If every vertex is eventually protected, then the searchers are successful. The minimum number of searchers required for success on G is called the deduction number of G.

Deduction was originally considered in the case that searchers could move at most once. In this talk, we allow each searcher to move k times, where k is a fixed positive integer. We give bounds on the k-move deduction number and consider similarities and differences with the one-move case. We discuss the process on various graph classes, including Cartesian and strong grids.

This is joint work with Nancy Clarke, Shannon Fitzpatrick and Melissa Huggan.

MACKENZIE CARR, Toronto Metropolitan University
Cooling Cartesian and lexicographic products of graphs

The cooling number of a graph measures the speed at which a slow-moving influence or contagion spreads on a graph. In this talk, we investigate the cooling number of two classical graph products: the Cartesian product and the lexicographic product. For the Cartesian product, we make use of isometric subgraphs of the factor graphs to determine families of graphs whose Cartesian products have largest possible cooling number in terms of their diameter. For the lexicographic product, we determine upper and lower bounds on the cooling number in terms of the cooling numbers of the factor graphs. This is joint work with Anthony Bonato, Caleb Jones, Trent Marbach, and Teddy Mishura.

NANCY CLARKE, Acadia University
Eternally Surrounding a Robber

We consider the Bodyguard Problem for graphs. This is a variation of Surrounding Cops and Robber but, in this model, a smallest possible group of bodyguards must surround a president and then maintain this protection indefinitely. We present

some general bounds on our parameter of interest, and pay particular attention to Cartesian, strong, and lexicographic products. We also give exact results for some common graph families. This is joint work with Danny Dyer and William Kellough.

ALEXANDER CLOW, Simon Fraser University
The Hole Story for Cop Number

The cop number of a graph, written $c(G)$, is a central invariant in the study of discrete-time processes on graphs. One advantage of cop number is its deep and well documented connection to concepts in structural graph theory, such as graph minors, subgraphs, and induced subgraphs. This talk will survey recent work on the connection between cop number and the structure of holes in a graph. A hole is an induced cycle of length at least 4. In doing so we will show that any graph G with $c(G) = \theta(G) = k \geq 2$, where $\theta(G)$ is the clique cover number, has C_ℓ as an induced subgraph for all $3 \leq \ell \leq k+1$. We will also prove that in any graph without holes of length at most t, $N[P]$ can be guarded by 1 cop for any short (as a function of t) induced path P, even when P is not a shortest path.

JULIEN CODSI, Princeton University

WILLIAM KELLOUGH, Memorial University of Newfoundland
Optimal Graph Burning Sequences and the Vertices They Contain

Graph burning is a process that models a social contagion spreading through a network. Many papers that study graph burning focus on how quickly the process can be completed on a given graph or a given family of graphs. In this talk, we instead focus on determining which vertices enable a social contagion to spread through a given graph in optimal time. We explore this idea for various families of graphs. We also demonstrate how to construct graphs that have relatively few vertices that the contagion can use to spread in optimal time. Joint work with Beth Ann Austin, Alfie Davies, Danny Dyer, and Logan Pipes.

ERIN MEGER, Queen's University

ARIANNE NANTEL, Queen's University

PAWEL PRALAT, Toronto Metropolitan University
Zombies on the Grid

In Zombies and Survivors, a set of zombies attempts to capture a lone survivor, Mindy, on a given connected graph. The game starts with zombies randomly choosing their initial locations. Then Mindy chooses her starting vertex, provided the game is played on a finite graph; for infinite grid $\mathbb{Z}^2$, we assume that Mindy starts at the origin. At each round, each zombie independently moves to a neighbouring vertex that minimizes the distance to Mindy; if there is more than one such vertex, one is chosen uniformly at random. (Multiple zombies may occupy one vertex.) Then Mindy moves to a neighbouring vertex or stays on her current vertex. The zombies win if eventually one of them eats Mindy by landing on her vertex (or if Mindy moves to an occupied vertex); otherwise, Mindy wins.

The zombie number $z(G)$ of a finite graph G is the minimum number of zombies needed to play such that the probability that they win, against Mindy's best strategy, is at least 1/2. For infinite grid $\mathbb{Z}^2$, we are interested in estimating the expected number of rounds Mindy can survive before she gets eaten.

In this talk, I present a few results for the game played on toroidal grids $T_n = C_n \square C_n$. I will also give bounds on the expected survival time for Mindy on the infinite grid $\mathbb{Z}^2$ with a Poisson process of zombies with density λ.

Joint work with Omer Angel and Peter Winkler.

BEN SEAMONE, Dawson College and Universite de Montreal

Discrete time-processes in online temporal graphs

A temporal graph is one where the presence of edges (or vertices) varies as a function of time. We consider a discrete model of temporal graphs where edges may appear or disappear at discrete time steps. In this talk, we will discuss recent research into the extension of familiar discrete-time processes from static graphs to temporal graphs. We will be particularly interested in the online setting, where the edges present at the next time-step are only revealed to the players at the end of the current time step.