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Stationary States on Periodic Metric Graphs with Nonzero Boundary Conditions

Stationary states of the nonlinear Schödinger (NLS) equation are considered on periodic metric graphs. Compared to the existing literature, we study stationary states with nonzero boundary conditions which model black solitons in optics. For a general graph with a single edge connecting compact cores symmetrically and periodically, we prove the existence of an energy minimizer pinned to the point of the edge symmetry. For the neckless graph example, we prove the coexistence of two energy minimizers pinned to two points of symmetry: one is in the middle of the link connected to two loops and the other one is in the middle of the loop connected to two links. In the semi-classical limit, which is equivalent to the limit of long edges, we prove that no other stationary states exist, whereas the link- pinned state is a local minimizer of energy and the loop-pinned state is a saddle point of energy without the pinning condition. We also prove that both are unstable in the time evolution of the NLS equation in such a way that the energy minimizer is strongly unstable with a real eigenvalue and the saddle point of energy is weakly unstable with a quadruplet of complex eigenvalues. The analytical results are illustrated away from the semi-classical limit by using numerical approximations.