Around harmonic analysis: applications to number theory, geometric measure theory, PDE, and more
Analyse harmonique: applications à la théorie des nombres, la théorie géometrique de la mesure, les EDP et plus
(Org: **Suresh Eswarathasan** (Dalhousie University) and/et **Chamsol Park** (University of Rochester))

DMITRY JAKOBSON, McGill
Nodal sets, negative eigenvalues and curvature prescription

We discuss conformal invariants arising fromspectra and eigenfunctions of conformally covariant differential operators, with applications to curvature prescription problems, as well as to the study of geometry of spaces of conformal structures.

KAINING JIA,
Uniform Quantitative Projection Theorems for Self Similar Sets

The classical Besicovitch–Federer projection theorem states that purely k-unrectifiable sets of finite k-dimensional Hausdorff measure have projections of zero measure in almost every direction. A quantitative problem is to determine how rapidly the measures of projected neighbourhoods decay, and if the estimates can hold uniformly.

In this talk, I will describe explicit self similar constructions exhibiting uniform logarithmic projection decay in arbitrary dimensions. For every $1 \leq k \leq d$ and $\alpha \in (0,1)$, we construct a finite homogeneous iterated function system satisfying the open set condition. The resulting attractor has positive and finite k-dimensional Hausdorff measure, while the k-dimensional measures of the projections of its δ-neighbourhood decay at the rate $O((\log(1/\delta))^{-\alpha})$, uniformly over all k-dimensional subspaces as $\delta \to 0$.

We also explore the relationship between Favard length and analytic capacity in the planar setting. For the planar self similar sets underlying our constructions, we establish a sharp logarithmic capacity law and a quantitative Vitushkin inequality with exponent 2α. These comparisons are superlinear for $\alpha > 1/2$, with exponents arbitrarily close to two. This is a joint work with Layna Mangiapanello, Caleb Marshall, and Victoria Risner.

DIMITRIS KOUKOULOPOULOS, Université de Montréal
Poissonian multiplicative chaos and divisor problems

The complex divisor function $\tau(n;\xi) = \sum_{d|n} d^{i\xi}$ is the Fourier transform of the sequence $\{\log d : d|n\}$. Its distribution shares many analogous properties to the Riemann zeta function: it is related to Poissonian multiplicative chaos, an object closely related to the Gaussian multiplicative chaos that rules the distribution of zeta. I will discuss the theory of the complex divisor function and study the distribution of its maximum value when ξ ranges over the interval $[1,2]$. This is based on ongoing joint work with Louis-Pierre Arguin and Paul Bourgade.

JUYOUNG LEE, University of British Columbia
The Fourier restriction problem for biradial functions

We determine the exact strong-type Fourier restriction range for the unit sphere $\mathbb{S}^{d-1}$ on the class of $O(d-k)\times O(k)$-invariant functions, for every $d \geq 4$ and $2 \leq k \leq d-2$. This includes the critical estimates beyond the biradial Stein–Tomas endpoint left open by Mandel and Oliveira e Silva. This is a joint work with Kinoshita, Oliveira e Silva, and Shiraki.

EMILY QUESADA-HERRERA, University of Lethbridge
Uncertainty, Fourier optimization, and the zeta function

Uncertainty principles, in their many qualitative and quantitative variants, state a limit on to what extent we can simultaneously control a function and its Fourier transform. Meanwhile, Fourier optimization problems impose restrictions on both, while aiming

to optimize a quantity of interest. Based on recent and ongoing work, we discuss some connections between these and several problems involving counting prime numbers, via the Riemann zeta function.

ERIC SAWYER, McMaster University

AI assistance in harmonic analysis: a probabilistic Fourier extension theorem on the Knapp line, and two weight inequalities

(1) Open AI has recently proved the diagonal Fourier extension theorem in all dimensions, but the conjecture remains open on the Knapp line. L. Korobenko and C. Rios and I, with help from Claude, have obtained a relatively short and simple proof of the probabilistic analogue on the Knapp line. The randomization eliminates the difficult Kakeya issues and so does not shed much light on the remaining deterministic conjecture, but does provide some evidence for it.

(2) B. Wick and I have recently made improvements in the known two weight norm inequalities for singular integrals that were aided by Claude. One particular case is that the two weight norm inequality for the Hilbert transform is now known to hold if and only if the two dual local testing conditions hold as well as the classical Muckenhoupt condition - improving the previous result of Lacey et al that used the larger tailed Muckenhoupt conditions. We will include a discussion of the interesting interactions with Claude and harmonic analysis in these two cases.

JOHN TOTH, McGill University

Non-concentration estimates for Laplace eigenfunctions on compact C^∞ manifolds with boundary

Let Ω be an n-dimensional compact Riemannian manifold with C^∞ boundary, and consider L^2-normalized eigenfunctions $-\Delta\phi_\lambda = \lambda^2\phi_\lambda$ with Dirichlet or Neumann boundary conditions. In this talk, I will describe reccent joint work with H. Christianson (UNC) on the extension of interior nonconcentration bounds up to the boundary. Specifically, using purely stationary local methods, we prove that for such Ω it follows that for *any* $x_0 \in \overline{\Omega}$ (including boundary points) and for all $\mu \geq C_\Omega h$ with sufficiently large constant $C_\Omega > 0$,

$$\|\phi_\lambda\|^2_{B(x_0,\mu)\cap\Omega} = O(\mu). \tag{1}$$

We also extend a result of C. Sogge to manifolds with smooth boundary and show that when $n \geq 3$,

$$\|\phi_\lambda\|_{L^\infty(\Omega)} \leq C\lambda^{\frac{n}{2}} \cdot \Big(\sup_{x\in\Omega} \|\phi_\lambda\|_{L^2(B(x,\lambda^{-1})\cap\Omega)}\Big). \tag{2}$$

The sharp sup bounds $\|\phi_\lambda\|_{L^\infty(\Omega)} = O(\lambda^{\frac{n-1}{2}})$ for Dirichlet or Neumann eigenfunctions proved by D. Grieser are then an immediate consequence of these bounds.

ALEXIA YAVICOLI, The University of British Columbia

The Erdős Similarity Conjecture and Rajchman measures

The Erdős similarity conjecture asserts that every infinite subset of the real line can be avoided by a measurable set of positive measure: the latter contains no translated and rescaled copy of the former. In this talk, I will prove the conjecture for sets supporting a Rajchman probability measure—that is, a probability measure whose Fourier transform tends to zero at infinity. More precisely, for any such set A and any $\varepsilon > 0$, we construct a closed, 1-periodic, nowhere dense set containing no affine copy of A, yet having measure at least $1 - \varepsilon$ in every interval of length 1.

This talk is based on a joint work with Natalia Mora Cuellar, Alex Iosevich, Neeraja Kulkarni and Ignacio Rojas Aravena.

Arxiv link: https://arxiv.org/abs/2609.04456

ZHICHUN ZHAI, MacEwan University

On Equivalent and Strengthened Affine Fractional Sobolev Inequalities

Affine fractional Sobolev inequalities provide refinements of classical fractional Sobolev inequalities by incorporating affine geometric structures. We establish several equivalent formulations of these inequalities. In particular, we show that affine fractional Sobolev inequalities, up to multiplicative constants, are equivalent to affine fractional Lorentz, Hardy, and isocapacitary inequalities.

We introduce affine fractional capacities and establish their fundamental properties, including capacitary strong-type inequalities. Using these capacities, we further strengthen the affine fractional Sobolev inequalities through embeddings into capacitary Lorentz spaces. As applications, we derive affine fractional Gagliardo–Nirenberg inequalities and affine fractional logarithmic Sobolev inequalities.

Our results highlight the interplay between harmonic analysis, affine geometry, fractional nonlocal regularity, and capacity theory, providing new perspectives on the refinement and equivalence of functional inequalities.