Algebraic combinatorics: longstanding questions in symmetric functions
Combinatoire algébrique: questions de longue date sur les fonctions symétriques
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HOUCINE BEN DALI, Harvard

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A (q,t)-Overview of q-analogs

By a simple homogenization process, one may turn a q-analog into a symmetric (q,t)-analog. Although simple, this bijective correspondence makes many concepts and identities become more natural, and proofs follow readily from classical properties of symmetric functions in two variables. A more intricate non-homogeneous extension is also developed. We illustrate this approach, revisiting recent developments in the study of γ-positivity, Lucas analogues, and the monoid of Cyclotomic generating functions. This also leads naturally to new constructions and conjectures. We further explore other avenues of investigations, included graded and equivariant γ-positivity and γ-anti-positivity (also known as alternatingly γ-positive).

HONG CHEN, UCLA
Characterizing Macdonald's Hypergeometric Series

Macdonald introduced multivariate hypergeometric series associated with Jack and Macdonald polynomials, extending the classical hypergeometric series and their zonal polynomial analogues arising in multivariate statistics. In this talk, we establish characterizations of these series by differential and q-difference equations. In the Jack setting, our results generalize earlier work on hypergeometric functions of matrix argument and subsequent developments of Yan and Kaneko. We also present q-difference characterizations in the Macdonald setting. These results provide a common perspective on hypergeometric series through the operators associated with Jack and Macdonald polynomials.

PATTY COMMINS, LACIM, Université du Québec à Montréal
Principal specializations of the t-chromatic symmetric function

We introduce the t-chromatic symmetric function, a variant of Stanley's chromatic symmetric function in which the variable t records the number of distinct colors used in a proper coloring. Although this t-variant is equivalent to the classical chromatic symmetric function as a graph invariant, its principal specializations retain substantially more information.

Our construction is motivated by the work of Bajo–Beck–Vindas-Meléndez on the q-chromatic polynomial and the conjecture of Loehr–Warrington that the nth principal specialization of the chromatic symmetric function distinguishes trees on at most n vertices, both of which stem from Stanley's longstanding question of whether the classical chromatic symmetric function distinguishes trees. We prove that the third principal specialization of the t-chromatic symmetric function distinguishes caterpillars with three spine vertices as well as elongated bistars. We conjecture, based on computational data, that the fourth principal specialization distinguishes all caterpillars.

This is joint work with Aram Bingham, Kyle Celano, Rosa Orellana, Jianping Pan, and Mary Claire Simone.

LUCAS GAGNON, USC

MEGUMI HARADA, McMaster University
Dimensions of type A Hessenberg varieties over a fixed sheet

This talk is on joint work with Martha Precup and Colleen Robichaux.

Hessenberg varieties $\mathcal{H}\mathrm{ess}(\mathsf{X}, \mathbf{h})$ are subvarieties of the flag variety parameterized by a Hessenberg function $\mathbf{h} : [n] \to [n]$ and a matrix $\mathsf{X} \in \mathfrak{gl}_n(\mathbb{C})$. Hessenberg varieties are known to be connected to the theory of quasisymmetric functions through the study of their cohomology rings. In recent work, Goldin and Precup showed the existence of flat degenerations of Hessenberg varieties to nilpotent Hessenberg varieties over the minimal sheet. This implies that all Hessenberg varieties over the minimal sheet have the same dimension. We generalize this dimension result to arbitrary sheets. Specifically, we prove that for a fixed Hessenberg function $\mathbf{h} : [n] \to [n]$, all Hessenberg varieties $\mathcal{H}\mathrm{ess}(\mathsf{X}, \mathbf{h})$ defined in the type A flag variety by linear operators X from the same sheet of the Lie algebra $\mathfrak{gl}_n(\mathbb{C})$ have the same dimension.

JOHN LENTFER, UCSD

JINTING LIANG, University of British Columbia
Symmetric quasisymmetric Schur-like functions

The area of quasisymmetric Schur-like functions had its genesis in quasisymmetric Schur functions, which naturally arose in the study of Macdonald polynomials and refined both Schur functions and their properties. One natural question is when are these quasisymmetric Schur-like functions symmetric. In this talk we classify when (row-strict) dual immaculate functions and (row-strict) extended Schur functions, as well as their skew generalizations, are symmetric. We also classify when their natural variants, termed advanced functions, are symmetric. In every case our classification recovers classical skew Schur functions. This is joint work with Maria Esipova and Stephanie van Willigenburg.

JIKE LIU, Rutgers University

ROSA ORELLANA, Dartmouth College
Proper implies improper for the chromatic symmetric function

Stanley's question of whether the chromatic symmetric function distinguishes trees has been unanswered for over thirty years. It has now been checked for trees with at most 29 vertices, and for several classes of trees. There is the perception that answering the question might be easier for the class of "proper" trees, i.e. trees where every vertex is either a leaf or adjacent to a leaf. In this abstract, we show that an affirmative answer for a subclass of proper trees, which we call "k-weighted", is equivalent to an affirmative answer for all trees. In addition, we show that in the class of k-weighted trees, the coefficients carry redundant information.

STEPHAN PFANNERER, Waterloo

SOPHIE REHBERG, UQAM

MARINO ROMERO, Mankato

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VASU TEWARI, University of Toronto Mississauga

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Honeycombs and Sums of Hermitian Matrices, Revisited

The Horn Problem is a classic problem, dating back to at least 1912. If A and B are two Hermitian matrices with known eigenvalue spectra, it asks about the possible spectra of $A+B$. The problem was fully resolved by Knutson, Tao, and Klyachko using tools from Schubert calculus and representation theory. An important component of the solution was honeycombs, a combinatorial model for Littlewood-Richardson coefficients developed by Knutson and Tao. Their key theorem states that a honeycomb with given boundary values exists if and only if there exists a triple $(A, B, A + B)$ of Hermitian matrices with corresponding eigenvalue spectra. This theorem gives a pleasant combinatorial and geometric answer to the original Horn problem. We present a new proof of this bridge between honeycombs and Hermitian matrices. The main surprise is that the proof can be made very elementary. It bypasses Schubert calculus and representation theory, making use only of linear algebra, basic calculus, and combinatorics. Our proof also suggests new perspectives on Littlewood-Richardson coefficients. This project is joint work with Ankur Moitra and Alex Postnikov.