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Kähler Manifolds and Chow's Theorem

The Whitney embedding theorem states that any smooth manifold has a smooth embedding into some Euclidean space. However, the holomorphic analogue fails; compact connected complex manifolds of positive dimension have no holomorphic embedding into $\mathbb{C}^n$. Instead, $\mathbb{CP}^n$ can replace $\mathbb{C}^n$ as an ambient space to embed such manifolds, but not all compact complex manifolds embed into $\mathbb{CP}^n$. We explain that the existence of such an embedding is constrained by vanishing cohomology in even degrees with real coefficients. By Chow's theorem, which we deduce from classical results of Serre, we present a correspondence between closed complex submanifolds and smooth projective algebraic subvarieties of $\mathbb{CP}^n$, further establishing that projective space is a natural ambient space to study such manifolds.