Symplectic Geometry with constraints from Algebraic Topology
Géométrie symplectique avec contraintes issues de la topologie algébrique
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TOM BAIRD, Memorial University of Newfoundland
Moduli spaces of stable bundles on a Hopf surface

A Hopf surface X is a non-Kaehler complex surface diffeomorphic to $S^1 \times S^3$. One can define a moduli space M_n parametrizing stable $SL_2(C)$-bundles over X of second chern number n. Using twisted Fourier-Mukai transforms and the Decomposition Theorem, we calculate the first and second betti numbers of M_n, answering a question posed by Witten in the context of string theory. This is joint work with Elizabeth Gasparim.

ANUNOY CHAKRABORTY, George Mason University
Character Varieties of RAAGs: Connectedness, Topology, and Dynamics

Right-angled Artin groups (RAAGs) interpolate between free groups and free abelian groups. For both of these extremes, and for closed surface groups, the outer automorphism group acts ergodically on the character variety with respect to a natural measure of full support. This talk asks whether that pattern continues for RAAGs in between.

We first show that the representation variety $\mathrm{Hom}(\Gamma_Q, G)$ is connected for every finite graph Q whenever the Lie group G satisfies a centralizer condition we call Property A, which includes $\mathrm{SU}(n)$ and $\mathrm{U}(n)$. We then focus on $\mathrm{SU}(2)$-character varieties of star-shaped RAAGs Γ_ℓ, whose graph is a tree with ℓ leaves around a single central vertex. Where the central generator maps to a non-central element, $\chi(\Gamma_\ell, \mathrm{SU}(2))$ is a trivial bundle $(-2, 2) \times (S^1)^\ell$ over the trace of the central generator. Outer automorphisms fix the central generator up to inversion, so every orbit stays in a single fiber. Hence, for $\ell \geq 2$, there is no $\mathrm{Out}(\Gamma_\ell)$-invariant ergodic measure of full support: the topology resembles the free and free abelian cases, but the dynamics does not.

We also show that $\chi(\Gamma_2, \mathrm{SU}(2))$ deformation retracts onto $S^3 \vee S^2 \vee S^2$ and that $\chi(\Gamma_\ell, \mathrm{SU}(2))$ is simply connected for every ℓ, supporting a conjecture of Biswas–Lawton–Ramras.

This is joint work with Sean Lawton, and with A. Bao, D. L. Duncan, J. Larson and K. McBride.

ADRIAN CHITAN, Western University

CHARLES DALY, Max Planck

MEGUMI HARADA, McMaster University
A construction of polyptych lattices via a pair of Gorenstein PL cones

This is joint work with Christopher Manon and Laura Escobar. The theory of polyptych lattices is motivated by a desire to simultaneously generalize phenomena in toric, cluster, and tropical geometry, among others. In particular, the theory builds a combinatorics-geometry dictionary reminiscent of that in toric geometry. The theory builds the notion of mutations directly into the definitions, and gives geometric interpretations to these mutations. In particular, in some cases, a polyptych-lattice ("PL") polytope encodes a compactification $\mathcal{X}$ of an affine variety $U = \mathrm{Spec}(\mathcal{A})$ whose coordinate ring $\mathcal{A}$ can be equipped with a valuation to a canonical semialgebra associated to the polyptych lattice. This is analogous to the known phenomenon that a

toric variety may be viewed as a compactification of a complex torus $(\mathbb{C}^*)^n$ by toric divisors, and the complex torus can be viewed as Spec of the Laurent polynomial ring, which can in turn be equipped with the valuation that takes a Laurent polynomial to its Newton polytope. Moreover, aspects of the geometry of the compactification $\mathcal{X}$ can be understood combinatorially; for instance, in some cases, the resulting compactifications are arithmetically Cohen-Macaulay, and have finitely generated class group and finitely generated Cox rings. The theory also naturally encodes multiple toric degenerations of the compactification $\mathcal{X}$ to toric varieties whose moment map images are related by mutations. In this talk, I recall the setup of this theory, and sketch a new construction of polyptych lattices coming from a pair of Gorenstein PL cones.

LISA JEFFREY, University of Toronto Scarborough

SHUO LIU, University of Maryland

CANSU OZDEMIR, University of Toronto
A Symplectic Look at İnönü's Large Group for Group Contractions

İnönü-Wigner group contractions were introduced in 1953 to explain how the symmetries of two systems connected by a limit in a parameter are related; for example the Poincaré group contracts to the Galilei group as the speed of light goes to infinity. In practice, a contraction is carried out on the Lie algebra $\mathfrak{g}$ of a Lie group G, by rescaling some of its generators with a parameter ε and taking the limit $\varepsilon \to 0$ of the Lie bracket. In 1956, answering a question of Cahit Arf, Erdal İnönü constructed a large group $L = \mathfrak{g} \rtimes_{\mathrm{Ad}} G$ that contains G together with its İnönü-Wigner contractions satisfying İnönü's condition (the rescaled generators span an $\mathfrak{h}$-invariant complement of the subalgebra $\mathfrak{h}$).

Recently, Khasanov and Kuperstein in 2011 described contractions through Lie algebra bundles; and Prinz, Schmeding and Schwartz in 2026 through germs of analytic curves. In this talk, we describe İnönü's construction of large algebra corresponding to the large group in both perspectives and show that every İnönü-Wigner contraction of $\mathfrak{g}$ is a subquotient of the large algebra $\mathfrak{l}$, the Lie algebra of L, and a subalgebra when İnönü's condition holds. Finally, for a contracted group K of G, we show that the phase spaces T^*G and T^*K of the original and the contracted systems both arise as symplectic reductions of T^*L, and discuss what this means for the corresponding Hamiltonian dynamics.

MARTIN PINSONNAULT, The University of Western Ontario
Representation stability for symplectomorphism groups of blow-ups of monotone rational surfaces

Let M be a monotone rational surface and $\tilde{M}_n$ its symplectic blow-up at n disjoint balls of equal small capacity. For $M = \mathbb{CP}^2$, the monotone $S^2 \times S^2$, and the monotone del Pezzo surfaces of degrees 4 to 8, I will explain why the rational cohomology of $B\mathrm{Symp}_h(\tilde{M}_n)$ forms, in each degree, a finitely generated FI-module, and is therefore uniformly representation stable as n varies. Taking S_n-invariants yields rational cohomological stability for the symplectomorphism groups permuting the exceptional divisors. The key geometric input is that the homotopy type of the space of symplectic embeddings of $n$disjoint balls is constant on an open polytope of small capacities. To our knowledge, these are the first representation-stability results for symplectomorphism groups. Joint work with Silvia Anjos.

PAUL SELICK, University of Toronto Scarborough

JONATHAN WEITSMAN, Northeastern University
What is Symplectic Geometry? The Weinstein "category" and the dual dg category

In Weinstein's symplectic "category", objects are symplectic manifolds and morphisms are canonical relations, which may not be composable; it is therefore only morally a category. We construct a true differential graded category which is morally dual to Weinstein's "category". The objects in this category are prequantum systems, associated to integral symplectic manifolds. The morphisms are a complex of differential forms twisted by a prequantum line bundle. We also consider the cohomology category, which turns out to be an (ordinary) linear category. In each case, Weinstein's morphisms are associated with currents dual to the forms. For the differential graded category, these are isotropic, or if preferred Lagrangian, currents, twisted by a section of the prequantum line bundle; in the case of cohomology, these currents are supported on *integral* Lagrangian submanifolds, equipped with a global covariant constant section of the prequantum line bundle. We then apply our methods to quantization, and show that for Kahler manifolds, a quantization in this category is related to holomorphic quantization. Along the way we show that in the case of prequantum systems, the Lefschetz action of $\mathfrak{sl}(2,R)$ on differential forms, studied by Brylinski, Mathieu, Guillemin, and Tseng-Yau in the symplectic case, is promoted to an action of the superalgebra $\mathfrak{osp}(1|2)$ on twisted differential forms, with $\mathfrak{sl}(2,R)$ as the even subalgebra. A first application of this superalgebra action is the vanishing theorem which shows the cohomology category is supported in middle dimension.

YUKAI ZHANG, University of Toronto

Complex Structure on the extended moduli space and the q-Hamiltonian space

We construct an almost complex structure on the extended moduli space and show that it is integrable. We then also construct an almost complex structure on the analogous q-Hamiltonian space and show that this structure is not integrable in any genus. We show that there exist other almost complex structures on the q-Hamiltonian space which are integrable on an open dense subset.