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Character Varieties of RAAGs: Connectedness, Topology, and Dynamics

Right-angled Artin groups (RAAGs) interpolate between free groups and free abelian groups. For both of these extremes, and for closed surface groups, the outer automorphism group acts ergodically on the character variety with respect to a natural measure of full support. This talk asks whether that pattern continues for RAAGs in between.

We first show that the representation variety $\mathrm{Hom}(\Gamma_Q, G)$ is connected for every finite graph Q whenever the Lie group G satisfies a centralizer condition we call Property A, which includes $\mathrm{SU}(n)$ and $\mathrm{U}(n)$. We then focus on $\mathrm{SU}(2)$-character varieties of star-shaped RAAGs Γ_ℓ, whose graph is a tree with ℓ leaves around a single central vertex. Where the central generator maps to a non-central element, $\chi(\Gamma_\ell, \mathrm{SU}(2))$ is a trivial bundle $(-2, 2) \times (S^1)^\ell$ over the trace of the central generator. Outer automorphisms fix the central generator up to inversion, so every orbit stays in a single fiber. Hence, for $\ell \geq 2$, there is no $\mathrm{Out}(\Gamma_\ell)$-invariant ergodic measure of full support: the topology resembles the free and free abelian cases, but the dynamics does not.

We also show that $\chi(\Gamma_2, \mathrm{SU}(2))$ deformation retracts onto $S^3 \vee S^2 \vee S^2$ and that $\chi(\Gamma_\ell, \mathrm{SU}(2))$ is simply connected for every ℓ, supporting a conjecture of Biswas–Lawton–Ramras.

This is joint work with Sean Lawton, and with A. Bao, D. L. Duncan, J. Larson and K. McBride.