Spectral Geometry
Géométrie spectrale
(Org: **Maxime Fortier Bourque** (Université de Montréal), **Alexandre Girouard** (Université Laval) and/et **Lawford Hatcher** (Université de Montréal))

SAMUEL AUDET BEAUMONT, Université Laval

THOMAS BECK, Fordham University
Nodal domain estimates for dumbbell domains

In this talk, we will discuss the nodal geometry of Neumann eigenfunctions of dumbbell domains, as the width of the thin neck joining the two sub-domains shrinks to zero. We will see that, generically, this forces the number of nodal domains to grow with the eigenvalue, and that in particular the third eigenfunction is Courant sharp (attains equality in the Courant nodal domain theorem) for sufficiently thin necks. In a symmetric case, we will also describe a criterion guaranteeing the existence of pairs of eigenfunctions that are Courant sharp. The proofs of these results require the leading order, non-zero, asymptotics of these eigenfunctions in each part of the dumbbell, which is established via a matched asymptotic analysis with logarithmic terms.

This is based on joint work with Yaiza Canzani, Andrew Lyons, and Jeremy Marzuola.

JARED BRONSKI, University of Illinois Urbana-Champaign

DMITRY FAIFMAN, Université de Montréal

JOSEF GREILHUBER, Massachusetts Institute of Technology

THOMAS HÉLIÈRE, Université Laval

DMITRY JAKOBSON, McGill
Small eigenvalues of degenerating Riemann surfaces

It is known that the small eigenvalues of the Laplacian of a Riemann surface close to the boundary of the modular space can be well approximated by the eigenvalues of the discrete Laplacian on a certain graph coming from the pair of pants decomposition of the surface. We provide a complete description of the sets of eigenvalues of the weighted graph Laplacian for all graphs on four vertices that correspond to a valid pair of pants decomposition of a surface of genus 3.

HANNA KIM, University of North Carolina at Chapel Hill
Extremal Metrics for GJMS operators in Critical Dimensions

GJMS operators are constructed by Graham, Jenne, Mason and Sparling as formally self-adjoint, elliptic, conformally covariant differential operators. In dimensions 2, the operator is Laplace-Beltrami operator and in dimensions 4, the operator is Paneitz operator. In critical dimensions, a symmetric stress energy tensor plays an important role in finding and characterizing extremal metric. Here, finding the extremal metric for k-th eigenvalue of the GJMS operator is essentially finding a local extrema among the analytic perturbation of the metric. In this talk, we discuss the necessary and sufficient conditions of extremal metrics of GJMS operators using the special properties of simplification in the critical dimensions.

RICHARD LAUGESEN, University of Illinois Urbana–Champaign
Rayleigh's conjecture for vibrating plates: simpler planar proof and nonlinear extensions

Which shape of clamped plate achieves the lowest fundamental tone? Rayleigh conjectured that among planar plates of fixed area, the disk minimizes the first eigenvalue of the bilaplacian. This fourth-order analogue of the Faber–Krahn theorem is substantially harder than the membrane problem, and was not proved in two dimensions until 1995, by Nadirashvili.

I will present a new and simpler proof, based on a paired comparison principle for Poisson equations with sign-changing data. The method extends naturally to a whole family of nonlinear clamped-plate problems: the disk minimizes the nonlinear Rayleigh quotient

$$\frac{\|\Delta u\|_{L^q(\Omega)}}{\|u\|_{L^p(\Omega)}}, \qquad u = \partial_\nu u = 0 \quad \text{on } \partial\Omega,$$

for every $1 \le p, q \le \infty$. Thus the original linear spectral conjecture opens the door to a much larger nonlinear universe.

(Joint work with Dorin Bucur.)

EGOR MOROZOV, Université de Montréal
Rotationally symmetric critical metrics for Laplace eigenvalues on tori in a conformal class

In 1996, El Soufi, Ilias, and Ros proved that the first normalized Laplace-Beltrami eigenvalue on torus in every rhombic conformal class is maximized by the flat metric. Surprisingly, the works of Petrides (2014) and Karpukhin (2025) show that the same assertion fails in non-rhombic conformal classes. This is supported by numerical experiments conducted by Kao, Lao, and Osting (2017), which suggest that the maximizing metric is rotationally symmetric. In this talk, I will construct explicitly a rotationally symmetric critical metric in each non-rhombic conformal class that is a natural candidate for maximizing the first eigenvalue in the conformal class (an AI-generated proof of the maximality is undergoing verification). A side effect of the construction is a new large family of $\mathbb{S}^1$-invariant harmonic 2-tori in the 3-sphere. The maps from this family are given explicitly in terms of elliptic integrals.

KSHITIJ PATIL, Simon Fraser University
Generalized Steklov-Helmholtz eigenvalues in the plane

For wave numbers in the first quadrant of the complex plane, we study the Steklov-Helmholtz eigenvalue problem on interior and exterior domains in the plane with Lipschitz boundaries. We show existence and discreteness of the eigenvalues in the case that the problem is not self-adjoint. High accuracy numerical computations lead to some conjectures related to the spectral geometry of the problem.

FRÉDÉRIC ROCHON, Université du Québec à Montréal
Vanishing in L^2-cohomology for some singular 3-Sasakian manifolds

After explaining what are 3-Sasakian manifolds and what type of singularities will be allowed, I will present a vanishing theorem in L^2-cohomology focusing on analytical aspects, notably about choices of self-adjoint extensions. This is a joint work with Panagiotis Dimakis.

EGOR SHELUKHIN, Université de Montréal

DAVID SHER, DePaul University
Heat trace and Riesz means with a large Robin parameter

We consider the Laplacian with Robin boundary conditions on a smooth Riemannian manifold with boundary and investigate the behavior of various spectral quantities as the Robin parameter approaches infinity. Although the spectrum, heat trace, and Riesz means converge pointwise to their Dirichlet analogues, the $asymptotics$ of these quantities as the spectral parameter approaches infinity, or as time approaches zero, do not. We investigate this phenomenon and explain the divergent behavior. Our main result is a two-term asymptotic expansion for the Robin heat trace that depends jointly on the time variable and the Robin parameter, with a remainder that satisfies a uniform bound independent of the Robin parameter. We apply this result to obtain a similar two-term asymptotic expansion for Riesz means.

ROMAIN SPECIEL, Stanford University

ALINA STANCU, Concordia University
A quantitative Ornstein–Uhlenbeck fundamental gap in the plane

For a bounded convex domain of diameter D, J. Sun and K. Wang proved in 2025 that the Dirichlet Ornstein–Uhlenbeck fundamental gap is bounded below by the gap on the centered interval of length D. I will present a quantitative refinement for origin-symmetric planar domains, where the excess over the interval gap is bounded below by $c_D w^6/D^3$, with w the minimum width. This gives a planar Gaussian counterpart to the width-dependent estimate of Amato, Bucur, and Fragalà for the Dirichlet Laplacian. Joint work with M. Farajje.

JOHN TOTH, McGill University
Non-concentration estimates for Laplace eigenfunctions on compact C^∞ manifolds with boundary

Let Ω be an n-dimensional compact Riemannian manifold with C^∞ boundary, and consider L^2-normalized eigenfunctions $-\Delta\phi_\lambda = \lambda^2\phi_\lambda$ with Dirichlet or Neumann boundary conditions. In this talk, I will describe reccent joint work with H. Christianson (UNC) on the extension of interior nonconcentration bounds up to the boundary. Specifically, using purely stationary local methods, we prove that for such Ω it follows that for *any* $x_0 \in \overline{\Omega}$ (including boundary points) and for all $\mu \geq C_\Omega h$ with sufficiently large constant $C_\Omega > 0$,

$$\|\phi_\lambda\|^2_{B(x_0,\mu)\cap\Omega} = O(\mu). \tag{1}$$

We also extend a result of C. Sogge to manifolds with smooth boundary and show that when $n \geq 3$,

$$\|\phi_\lambda\|_{L^\infty(\Omega)} \leq C\lambda^{\frac{n}{2}} \cdot \Big(\sup_{x\in\Omega} \|\phi_\lambda\|_{L^2(B(x,\lambda^{-1})\cap\Omega)}\Big). \tag{2}$$

The sharp sup bounds $\|\phi_\lambda\|_{L^\infty(\Omega)} = O(\lambda^{\frac{n-1}{2}})$ for Dirichlet or Neumann eigenfunctions proved by D. Grieser are then an immediate consequence of these bounds.

DENIS VINOKUROV, Université de Montréal
Eigenvalue optimisation via a first-variation formula

Optimization problems often become easier to solve when one allows more singular objects. In eigenvalue optimization, such a relaxation can lead to operators with nonempty essential spectrum and eigenvalues lying at its edge. In this talk, I will present a general approach to deriving extremality conditions in this setting, using weighted Laplace and Steklov eigenvalues as examples. Allowing singular weights leads to a short proof of the existence of maximizers. The resulting extremality conditions, in turn, improve the a priori regularity of maximizers and lead to a geometric description of all maximizing weights via harmonic maps into spheres and free boundary harmonic maps into balls, respectively.