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Rotationally symmetric critical metrics for Laplace eigenvalues on tori in a conformal class

In 1996, El Soufi, Ilias, and Ros proved that the first normalized Laplace-Beltrami eigenvalue on torus in every rhombic conformal class is maximized by the flat metric. Surprisingly, the works of Petrides (2014) and Karpukhin (2025) show that the same assertion fails in non-rhombic conformal classes. This is supported by numerical experiments conducted by Kao, Lao, and Osting (2017), which suggest that the maximizing metric is rotationally symmetric. In this talk, I will construct explicitly a rotationally symmetric critical metric in each non-rhombic conformal class that is a natural candidate for maximizing the first eigenvalue in the conformal class (an AI-generated proof of the maximality is undergoing verification). A side effect of the construction is a new large family of $\mathbb{S}^1$-invariant harmonic 2-tori in the 3-sphere. The maps from this family are given explicitly in terms of elliptic integrals.