**Representation Theory**
**Théorie des représentations**
(Org: **Alistair Savage** (University of Ottawa) and/et **Alex Weekes** (Université de Sherbrooke))

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**JON BORETSKY**, McGill University
*Subdivisions of MV polytopes and permutohedra*

Mirković–Vilonen (MV) polytopes are encoded combinatorially by Berenstein–Zelevinsky (BZ) data. BZ data satisfy the tropical Plücker relations, imposed by the geometry of the affine Grassmannian, together with additional inequalities ensuring the resulting polytope is geometrically realizable. We introduce active intervals of an MV polytope, which are sets of vertices whose convex hull is a translated scaled Bruhat interval polytope. This generalizes the active diagonals of rank 2 MV polytopes. In type $A$, the tropical Plücker relations cut out the positive complete flag Dressian, so BZ data are points of the positive Dressian. We leverage the connection between the positive Dressian and coherent subdivisions of the permutohedron to show that a type $A$ GGMS polytope (that is, a generalized permutohedron) is an MV polytope if and only if the convex hulls of its active intervals subdivide it. We also show that, as a point of the Dressian, the BZ data induce the coherent subdivision of the permutohedron indexed by the same intervals.

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**EMILY CLIFF**, Université de Sherbrooke
*Constructing generic bricks from quasi-universal sheaves on the moduli space of quiver representations*

Let $A$ a finite-dimensional algebra, presented as the quotient of the path algebra of a quiver. An $A$-module $M$ is called a *brick* if its endomorphism ring is a division algebra, and $A$ is said to be *brick-infinite* if it admits infinitely many isomorphism classes of finite-dimensional bricks. Mousavand conjectures that if $A$ is brick-infinite, then there exists infinitely many isomorphism classes of bricks of the same dimension vector. It is also expected that this infinite family of finite-dimensional bricks should be controlled by a single, but infinite-dimensional brick, called a *generic brick*.

In this talk, I will explain the geometry underlying the relationship between these notions. Indeed, in the case that we have a moduli space $\mathcal{M}$ of stable $A$-modules of a given dimension vector, we construct a so-called *quasi-universal* sheaf on $\mathcal{M}$. We use this sheaf to build an infinite-dimensional brick at the generic point, while its fibres over closed points give an infinite family of finite-dimensional bricks.

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**PATTY COMMINS**, LACIM, Université du Québec à Montréal
*The combinatorial representation theory of the Mantaci–Reutenauer algebras*

The Mantaci–Reutenauer algebra is a finite-dimensional algebra arising as a subalgebra of the hyperoctahedral group algebra. It is a type B analogue of Solomon's descent algebra, a well-studied subalgebra of the symmetric group algebra with connections to algebraic combinatorics. Motivated by questions of Bonnafé, we study its representation theory, determining its (graded) Cartan matrix, Ext-quiver, and representation type. We use the combinatorics of signed necklaces and bipartitions to study these representation-theoretic invariants, and we recover the corresponding known results for Solomon's descent algebra as a special case. This is joint work with Alice Dell'Arciprete.

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**IVAN DIMITROV**, Queen's University
*$U(h)$-free modules over the Lie superalgebras $sl(m|n)$*

In this talk I will discuss the existence and classification of certain classes of $sl(m|n)$-modules which when restricted to the universal enveloping algebra $U(h)$ of a Cartan subalgebra h are free of finite rank. Particular results include: Classification of modules of rank 2; existence and structure of modules of higher rank; weight modules obtained from $U(h)$-free modules via the weighting functor.

**MONICA GARCIA**, LACIM
*Infinite superfriezes*

A frieze is an infinite arrangement of integers that satisfies the simple unimodular rule $ad - bc = 1$. Since their introduction by Coxter, friezes have become an important topic in algebraic combinatorics due to their connections to cluster algebras, triangulations of surfaces and perfect matchings. In this talk, I will introduce the notion of a superfrieze: a generalization of friezes where the entries are elements of a supercommutative algebra, and recall some of their geometric and combinatorial realizations. I will report on joint work with A. Burcroff, İ. Çanakçı, F. Fedele and V. Klász on how to construct infinite superfriezes from decorated skeletal triangulations of annuli.

**NICOLLE GONZÁLEZ**, University of British Columbia
*Sign components of bosonic-fermionic coinvariant rings*

Bosonic-fermionic coinvariant rings $R_n^{(k,\ell)}$ arise as the quotients of polynomial ring tensored an exterior algebra by the ideal of symmetric group invariants under the so-called diagonal action. These rings generalize the diagonal coinvariant rings $R_n^{(2,0)}$ studied by Garsia and Haiman in the 1990's and early 2000's, whose characters were at the core of the celebrated shuffle theorems in algebraic combinatorics. Although the rings $R_n^{(2,0)}$ satisfy some remarkable properties, many of these rely on geometric arguments not available for general $R_n^{(k,\ell)}$. In this talk, I will discuss a purely algebraic approach to computing the sign isotypic components of these rings and some algebraic and combinatorial consequences that follow from this.

**NIKITA GRYGORYEV**, Perimeter Institute/University of Waterloo
*Line defects, CoHA-bimodules, and Monoidal Cluster Categorification*

Line operators in a four-dimensional $\mathcal{N} = 2$ quantum field theory are expected to form a graded rigid monoidal category equipped with (the leading pole part of) a meromorphic braiding, and whose Grothendieck ring is a quantum cluster algebra; the coherent Satake category of Cautis–Williams is the model example. I will describe a construction of such categories from cluster-like combinatorial data: to a seed $\Theta$ (a quiver $Q$ together with a choice of charge lattice) it attaches a monoidal category $\mathcal{C}_\Theta$ of bimodules over the cohomological Hall algebra of $Q$, generated by sub-bimodules of the cohomology of stacks of framed representations (taken internally to the IR-category $\mathcal{C}_\Theta^{IR}$, a categorified quantum torus on the charge lattice). I will try to convey the representation-theoretic patterns in $\mathcal{C}_\Theta$, some proven and some conjectural: analogues of standard, costandard and simple objects, freeness over the Hall algebra, character maps to the quantum torus related by quantum dilogarithms, meromorphic $R$-matrices, and short exact sequences categorifying mutation. For the Kronecker quiver, $\mathcal{C}_\Theta$ should categorify the quantized $K$-theoretic Coulomb branch of pure gauge theory, with $\Theta$ distinguishing the gauge groups $SL_2$, $PGL_2$ and $GL_2$. Based on joint work with D. Gaiotto and W. Li (arXiv:2406.07134) and on work in progress.

**JOEL KAMNITZER**, McGill
*Skein algebras of punctured spheres, Coulomb branches and knot homology*

Certain skein algebras of surfaces are expectied to be isomorphic to K-theoretic Coulomb branch algebras. This suggests that these algebras can be categorified using coherent sheaves on BFN spaces. I will explain how we can combine my older work with Cautis into this perspective in order to obtain homological knot invariants for knots in thickened punctured spheres.

**NICCOLO MATTAREI**, University of Ottawa
*Double affine Frobenius Hecke algebras*

In this talk, I will discuss a version of the $\mathrm{GL}_n$ double affine Hecke algebra (DAHA) that is decorated by a symmetric Frobenius superalgebra. This is a double affine version of Rosso and Savage's *quantum affine wreath algebras*. When the Frobenius algebra is the group algebra of a finite cyclic group, we obtain new double affine versions of the affine Yokonuma–Hecke

algebras. I will explain how the diagrammatic interpretation of DAHA using braids on a torus motivates the presentations, basis theorem and symmetries such as the $\mathrm{SL}_2(\mathbb{Z})$-action by outer automorphisms on this more general version.

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**THOMAS MORAN**, University of Ottawa
*Tensor product affine wreath superalgebras*

To every Frobenius superalgebra, we associate a family of superalgebras that we call tensor product affine wreath superalgebras. They can be viewed both as a tensor product version of Savage's affine wreath superalgebras, and as a degenerate Frobenius version of the tensor product affine Hecke algebras defined by Webster, Maksimau, and Stroppel. We establish the basic structure theory of these new superalgebras, including a basis theorem and a description of the center. Furthermore, in the Clifford setting, we relate them to the tensor product degenerate spin affine Hecke superalgebras via a Morita superequivalence.

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**MONICA NEVINS**, University of Ottawa
*Representations of p-adic groups near the identity*

The infinite-dimensional irreducible representations of p-adic groups have an elegant geometric structure near the identity. The analytic nature of this structure, expressed in terms of the character of the representation, has been known since the 1970s, and recently progress has been made on understanding it in more concrete terms via the restriction of representations to sufficiently small compact open subgroups. In this talk, we describe the ideas and the techniques involved, focusing on geometric and combinatorial aspects. Parts of this talk are based on ongoing joint work with Ekta Tiwari (IISER Pune), with Zander Karaganis (UBC), and with David Schwein (Utah).

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**THEO PINET**, McGill

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**ANDREW RIESEN**, M.I.T.
*Associated Varieties of Finite-Group Orbifolds*

In this talk, I will discuss how the associated variety of a vertex algebra behaves under finite-group orbifolds. In particular, under suitable assumptions, the associated variety of $V^G$ is isomorphic to $X_V/G$ as a reduced Poisson variety. This has several nice consequences, such as the descent of strong rationality: if $V$ is strongly rational, then so is $V^G$, and the category of $V^G$-modules is therefore a modular tensor category.

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**HOMA ROUDINI**, University of Ottawa
*Explicit Classification of Irreducible Real Modules of Real Forms of* $\mathfrak{gl}(m|n)$

Sahi, Salmasian, and Serganova established a general framework for classifying finite-dimensional irreducible real modules of a broad class of real Lie superalgebras, reducing the problem to the study of parity and conjugation functors on irreducible complex modules.

Building on their work, we obtain explicit classification criteria for various real forms of $\mathfrak{gl}(m|n)$. In particular, we describe how highest weights determine the possible endotypes of irreducible real modules.

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**LEONID RYBNIKOV**, Université de Montréal
*Cactus groups, tableaux and Bethe monodromy*

The cactus group plays the role of the braid group in the theory of Kashiwara crystals. It is the equivariant fundamental group of the real locus of the Deligne–Mumford moduli space of stable rational curves, where the symmetric group permutes

all marked points except one. I will discuss two developments connecting this geometry with Young tableaux and quantum integrable systems.

First, I will introduce the mirabolic cactus group, a new group defined using two distinguished marked points, and describe its action on semistandard skew Young tableaux. The action combines Bender–Knuth and partial Schützenberger involutions and arises as the monodromy of Bethe eigenlines for the XXX Heisenberg spin chain. This gives a geometric explanation of the relationship between the Berenstein–Kirillov and cactus groups.

I will then discuss the big cactus group, in which all marked points are treated symmetrically. This is a geometrically more natural but substantially less studied object. We give a presentation of this group and show that its relations hold on rectangular tableaux and invariant tensors in crystals of arbitrary semisimple type.

The talk is based on joint work with Anfisa Gurenkova and Iva Halacheva, and with Gabriel Frieden and Pavlo Pylyavskyy.

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**HADI SALMASIAN**, University of Ottawa
*Quantum Weyl algebras and idempotented Howe duality*

I will explain a variant of quantized Howe duality which connects with quantum invariant theory through the lens of q-deformed Weyl algebras. This is based on a joint work with S. Sahi and M. Yakimov.

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**JAKE S. ZIMMERMANN SIMMONS**, University of Toronto
*Regular Bisections of Nilpotent Subalgebras of $\mathfrak{sl}(n)$*

In this talk, we explore how nilpotent regular Lie subalgebras $\mathfrak{g}$ of the special linear algebra $\mathfrak{sl}(n)$ can be split into bisections, pairs of subalgebras of $\mathfrak{g}$ that equal $\mathfrak{g}$ as a vector space direct sum. These splittings help reveal internal structure of the $\mathfrak{g}$ and provide a concrete way to understand how its parts fit together, through a matrix representation of the bisections. There is a classical bijection between biclosed subsets of root systems and biclosed subsets of root systems. In this talk we will investigate a counting of bisections of the strict upper triangle algebra $\mathfrak{n}(n) \subseteq \mathfrak{sl}(n)$ which provides a new perspective on this bijection, and will discuss how this could lead to investigations using the Weyl group of a matrix representations of bijections, implicitly giving a group structure on all bisections any nilpotent regular subalgebra of $\mathfrak{sl}(n)$. There will be $n!$ bisections of $\mathfrak{n}(n)$ and this will be proven. The proof uses an induction argument that introduces the idea of extending algebras, and those extensions being complementary to each other. Only the bisections of some algebras can be represented as matrices, but all are relevant to the Weyl group as structure of bisections in general can be inherited by those that can be represented as matrices. Lastly, we present an algorithm, which we will call the falling terms algorithm, to count bisections of subalgebras of the strict upper triangle.

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**JOSHUA SUSSAN**, Medgar Evers College
*p-DG structures in link homology*

The state space of a gl(2)-web may be equipped with a p-DG structure. This construction may be extended to the Khovanov homology of a link. As a consequence, we obtain a categorification of the Jones polynomial at roots of unity.

This is joint work with Y. Qi, L.H. Robert, and E. Wagner.

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**ALEKSANDR (SASHA) TRUFANOV**, Université de Montréal
*Athinization of irreducible $\widehat{\mathfrak{gl}}_n$-modules with dominant highest weights*

We study the Gelfand–Tsetlin realization of generic Verma modules for the affine Lie algebra $\widehat{\mathfrak{gl}}_n$ by viewing them as thin modules over the affine Yangian $Y(\widehat{\mathfrak{sl}}_n)$. By the results of [FFNR11], these modules admit a basis indexed by periodic Gelfand–Tsetlin patterns with explicit formulas for the Yangian action, and we identify them with the evaluation modules introduced by Kodera [K21]. Our main result describes the specialization from generic highest weights to dominant highest weights (not necessarily integral). We call the resulting construction athinization: an irreducible $\widehat{\mathfrak{gl}}_n$-module, which is not thin as a module over the

affine Kac–Moody algebra, is realized as a thin module over the larger (and "more affine") algebra $Y(\widehat{\mathfrak{sl}}_n)$. Combinatorially, this realization is obtained by restricting the generic periodic Gelfand–Tsetlin basis to a distinguished subset of permitted patterns. We prove that the span of these patterns carries a well-defined affine Yangian action. In particular, this construction yields explicit Gelfand–Tsetlin-type bases for admissible representations of $\widehat{\mathfrak{gl}}_n$ in the sense of Kac–Wakimoto, providing a new combinatorial realization of these modules. We compare the formulas for characters coming from this combinatorics with those for minimal models of $W$-algebras of type $A_n$ via the principal specialization. Further, we obtain analogous results for representations of $U_q\widehat{\mathfrak{gl}}_n$ via their realization as thin modules over the quantum toroidal algebra of $\mathfrak{gl}_n$. Based on work https://arxiv.org/abs/2607.07653 joint with M. Bershtein, E. Mukhin and L. Rybnikov.

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**BEN WEBSTER**, University of Waterloo/Perimeter Institute
*Centers of cyclotomic Hecke algebras*

Cyclotomic Hecke algebras (or Ariki-Koike algebras) are a natural class of generalizations of the Hecke algebras of types A and B. Despite decades of study, one of the most basic questions about them has remained open: what is their center? There are natural generalizations of the Jucys-Murphy elements, whose symmetric polynomials are obviously central; it has long been expected that all central elements have this form. While straightforward when the algebra is semi-simple, and known from work of Francis-Graham and Brundan in many special cases, at arbitrary parameters it proved surprisingly resistant to proof. I'll discuss a recent proof (joint with Jun Hu, Andrew Mathas and Lei Shi) that confirms this conjecture. The key input is geometric: an isomorphism between the centers of blocks and the cohomology of quiver varieties of affine type A, which intertwines the action of a "loop algebra."

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**MERI ZAIMI**, Perimeter Institute
*Knot contact homology as a planar limit of Chern-Simons theory*

In the context of holographic duality in physics, it is expected that the quantum knot invariants obtained from $SU(N)$ Chern-Simons theory in the planar limit ($N \to \infty$, $q \to 1$ and $q^N$ fixed) are related to knot contact homology, and hence to augmentation varieties. In particular, it is conjectured that the augmentation polynomial of a knot is the classical limit of a deformed $\hat{A}$-polynomial, which describes recursions for colored HOMFLYPT polynomials. In this talk, I will discuss how to study this connection via the "HOMFLYPT difference module", which encodes the relations among the HOMFLYPT polynomials with different antisymmetric colorings. The construction relies on a spider category for $U_q(gl_N)$ where, in addition to the usual objects carrying integer labels, associated to the antisymmetric powers of the standard representation, we include objects carrying formal labels, which allow the color to be treated as a variable. The main result is that the classical limit of the HOMFLYPT difference module is isomorphic to the degree 0 part of the abelianized knot contact homology. The conjecture relating the augmentation polynomial to the deformed $\hat{A}$-polynomial can be reformulated in terms of certain algebraic properties of the module. This is based on work with Ben Webster.

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**KIRILL ZAYNULLIN**, University of Ottawa
*The unimodular degree and the w-index of a Kac-Moody root system*

We introduce combinatorial analogues (the unimodular degree and the $w$-index) of two celebrated geometric invariants of a split simple simply connected linear algebraic group $G$ to the Kac–Moody setting: the canonical dimension of $G$ and the Grothendieck torsion index of $G$.

We prove finiteness of the unimodular degree for an arbitrary generalized Cartan matrix using coinvariant algebras over finite fields. In rank two, we determine the unimodular degree and obtain a product formula for all $w$-indices in terms of recursively defined dihedral polynomials. Finally, we use the $1$-chains (type A) and $2$-chains (type C) to obtain explicit lower bounds for all untwisted and twisted affine types. This is a joint work with Edward Richmond.