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Line defects, CoHA-bimodules, and Monoidal Cluster Categorification

Line operators in a four-dimensional $\mathcal{N} = 2$ quantum field theory are expected to form a graded rigid monoidal category equipped with (the leading pole part of) a meromorphic braiding, and whose Grothendieck ring is a quantum cluster algebra; the coherent Satake category of Cautis–Williams is the model example. I will describe a construction of such categories from cluster-like combinatorial data: to a seed Θ (a quiver Q together with a choice of charge lattice) it attaches a monoidal category $\mathcal{C}_\Theta$ of bimodules over the cohomological Hall algebra of Q, generated by sub-bimodules of the cohomology of stacks of framed representations (taken internally to the IR-category $\mathcal{C}_\Theta^{IR}$, a categorified quantum torus on the charge lattice). I will try to convey the representation-theoretic patterns in $\mathcal{C}_\Theta$, some proven and some conjectural: analogues of standard, costandard and simple objects, freeness over the Hall algebra, character maps to the quantum torus related by quantum dilogarithms, meromorphic R-matrices, and short exact sequences categorifying mutation. For the Kronecker quiver, $\mathcal{C}_\Theta$ should categorify the quantized K-theoretic Coulomb branch of pure gauge theory, with Θ distinguishing the gauge groups SL_2, PGL_2 and GL_2. Based on joint work with D. Gaiotto and W. Li (arXiv:2406.07134) and on work in progress.