LEONID RYBNIKOV, Université de Montréal

Cactus groups, tableaux and Bethe monodromy

The cactus group plays the role of the braid group in the theory of Kashiwara crystals. It is the equivariant fundamental group of the real locus of the Deligne–Mumford moduli space of stable rational curves, where the symmetric group permutes all marked points except one. I will discuss two developments connecting this geometry with Young tableaux and quantum integrable systems.

First, I will introduce the mirabolic cactus group, a new group defined using two distinguished marked points, and describe its action on semistandard skew Young tableaux. The action combines Bender–Knuth and partial Schützenberger involutions and arises as the monodromy of Bethe eigenlines for the XXX Heisenberg spin chain. This gives a geometric explanation of the relationship between the Berenstein–Kirillov and cactus groups.

I will then discuss the big cactus group, in which all marked points are treated symmetrically. This is a geometrically more natural but substantially less studied object. We give a presentation of this group and show that its relations hold on rectangular tableaux and invariant tensors in crystals of arbitrary semisimple type.

The talk is based on joint work with Anfisa Gurenkova and Iva Halacheva, and with Gabriel Frieden and Pavlo Pylyavskyy.