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Subdivisions of MV polytopes and permutohedra

Mirković–Vilonen (MV) polytopes are encoded combinatorially by Berenstein–Zelevinsky (BZ) data. BZ data satisfy the tropical Plücker relations, imposed by the geometry of the affine Grassmannian, together with additional inequalities ensuring the resulting polytope is geometrically realizable. We introduce active intervals of an MV polytope, which are sets of vertices whose convex hull is a translated scaled Bruhat interval polytope. This generalizes the active diagonals of rank 2 MV polytopes. In type A, the tropical Plücker relations cut out the positive complete flag Dressian, so BZ data are points of the positive Dressian. We leverage the connection between the positive Dressian and coherent subdivisions of the permutohedron to show that a type A GGMS polytope (that is, a generalized permutohedron) is an MV polytope if and only if the convex hulls of its active intervals subdivide it. We also show that, as a point of the Dressian, the BZ data induce the coherent subdivision of the permutohedron indexed by the same intervals.