Nonlinear Waves and Dispersive PDEs: Recent Advances in Theory and Computations
Ondes non-linéaires et EDP dispersives: Avancées récentes théoriques et calculatoires
(Org: **Yvonne Alama Bronsard** (MIT) and/et **Maria Ntekoume** (Concordia University))

MARCO BERTOLA, Concordia University
Zakharov Shabat Spectra and Minimal Dirichlet Energy

In this talk (joint work with A. Tovbis) I will discuss the spectra of finite-gap solutions for fNLS We will show that the average intensity is equivalent to a problem in harmonic analysis of computing a specific Dirichlet energy associated with a domain in the upper half plane. We will also show that the Zakharov-Shabat spectra of finite gap solutions realize (local) minima of these Dirichlet energies amongst a well defined class of compact sets, a result that suggests a local stability property. Some considerations related to so-called "soliton gas condensates" will be discussed.

YVONNE ALAMA BRONSARD, Nantes Université & CNRS
From numerical computations to wave kinetic theory for the Benjamin—Ono equation

We introduce a new approach for solving the Benjamin—Ono equation as well as other integrable PDEs, based upon recent theoretical breakthroughs on explicit formulas for these equations. It opens the way to numerical approximations which are far more accurate and efficient for simulating these equations, allowing for the study of the asymptotic behavior of the solutions. Moreover, in the setting of randomized initial data, by rescaling and iterating this explicit formula, combined with matrix concentration inequalities, we obtain insight on the Wave Kinetic theory for BO, up to the physically relevant kinetic time scale.

MATTHIEU CADIOT, Ecole Polytechnique
Overhanging waves in the water wave equations with constant vorticity

In this talk, we present a computer-assisted methodology for proving the existence of periodic traveling gravity waves at the free surface of water, in a flow of constant vorticity over a flat bed. Using conformal mappings, the free-boundary problem is reformulated as a quasilinear pseudodifferential equation for a periodic function of a single variable. We then expand the solution in a Fourier series, reducing the problem to an infinite-dimensional system of equations for the Fourier coefficients.

To establish existence, we employ a Newton–Kantorovich type argument, proving the existence of a true solution in a neighborhood of a numerically computed approximation. The verification of the hypotheses of this fixed-point argument relies on a careful combination of analytical estimates and rigorous numerical computations.

This approach enables us to rigorously prove the existence of an overhanging wave, that is, wave profiles that are no longer graphs of functions. Moreover, it allows for the construction of branches of solutions parameterized by the relative mass flux, bifurcating from the trivial flat solution. In particular, we rigorously construct a branch that originates at the flat solution and extends to a wave with a self-intersecting profile, passing through overhanging configurations along the way.

This is a joint work with Susanna Haziot (Princeton).

JOEL DANHE, University of Minnesota
Non-uniqueness for the complex Ginzburg–Landau equation

We show that singularities of the three-dimensional cubic complex Ginzburg–Landau equation developing from smooth initial data can induce non-uniqueness of solutions after the time of blowup. The solutions we study start as backward self-similar solutions, and the source of the non-uniqueness is an unstable eigenvalue of the linearization around the forward self-similar profile generated by the singularity. Using a computer-assisted proof, we rigorously establish the existence of this unstable eigenvalue for specific parameter values in the equation. The work is in part motivated by similarities to the Navier–Stokes

equations, with which the complex Ginzburg–Landau equation shares key features such as scaling symmetry and an energy inequality, where recent breakthroughs on finite-time blowup from smooth initial data make the question of post-blowup non-uniqueness highly relevant.

STEPHEN GUSTAFSON, University of British Columbia
Internal modes of solitons of degenerately near-cubic NLS in 1D

For nonlinear dispersive PDE, the presence (or absence) of internal modes in the spectrum of the linearized operator around a soliton solution has implications for the long-time dynamics of general solutions close to the soliton (for its asymptotic stability, for example). Recent work of Martel, and of Rialland, studied the internal modes of solitons of near-cubic nonlinear Schrodinger equations in one dimension, and established their asymptotic stability. Here we extend this analysis of internal modes to a class of degenerate perturbations of the cubic not considered there, and realize this situation for some combined-power nonlinearities. This is joint work in progress with Yvan Martel.

SLIM IBRAHIM, University of Victoria
Orbital stability of dark solitons for the intermediate nonlinear Schrödinger equation

The intermediate nonlinear Schrödinger equation models the envelope of weakly nonlinear internal waves in a stratified fluid of finite depth. It admits two families of dark solitons, traveling waves that appear as localized dips in intensity on a nonzero background. In this talk, I will show that both families are orbitally stable in two regimes. The first occurs when the fluid is sufficiently shallow and the soliton width is bounded below. The second occurs at large depth, provided the soliton is sufficiently wide compared with the depth. In either regime, a small perturbation stays close to the soliton for all time, up to translation and phase rotation.

The proof explains why a shallow-water approximation remains useful even at large depth. In hydrodynamic variables, stability reduces to a spectral question for a nonlocal operator. On the scale of the soliton, the comparison operator is the familiar linearization about a Korteweg–de Vries solitary wave.

This is joint work with Takafumi Akahori and Nobu Kishimoto.

JEAN-PHILIPPE LESSARD, McGill University
Computer-Assisted Proofs of Localized Patterns in the Planar Swift-Hohenberg Equation

The Swift-Hohenberg equation is a classical model for pattern formation, exhibiting a rich variety of spatially periodic and localized structures. While numerical simulations suggest the existence of intricate localized patterns in two spatial dimensions, establishing their existence rigorously remains challenging. In this talk, I will present a computer-assisted approach for proving the existence of smooth, stationary, non-radially symmetric localized solutions of the planar Swift-Hohenberg equation. Our method combines a Newton-Kantorovich argument with Fourier analysis, analytic estimates, and rigorous numerical computations to validate solutions on the unbounded plane. We also establish that these localized patterns arise as limits of families of spatially periodic solutions as their periods tend to infinity. This is joint work with Matthieu Cadiot (École Polytechnique, France) and Jean-Christophe Nave (McGill).

JEAN-CHRISTOPHE NAVE, McGill University
The Characteristic Mapping Method: from Dissipative to Dispersive Problems

The characteristic mapping method represents a solution by composing short-time maps. In joint work with Holman–Bissegger and Taylor (arXiv:2611.08261), we extended this construction to dissipative equations by pairing each map with a weight that accounts for local compression. The map-weight pairs compose by a semidirect product, and the resulting field can be evaluated directly off grid.

I will describe how this formulation extends to periodic equations of the form $u_t = Au - \lambda\partial_x(u^2/2)$, where A is a dissipative or dispersive Fourier multiplier. The map is advanced by semi-Lagrangian tracing, while an exponential integrator accounts for

the phase of each resolved linear mode. Numerical examples will illustrate the influence of dispersion on phase accuracy and the resolution of fine spatial scales.

I will conclude with the role that geometry plays in the analysis of the method and discuss the additional structure available when the underlying equation admits an Euler-Arnold formulation. Time permitting, I will also discuss a connection with the work of Alama-Bronsard and Laurens on the Benjamin-Ono equation.

DMITRY PELINOVSKY, McMaster University
Nonlinear stability of periodic waves in the Benjamin-Ono equation

We construct the Lyapunov functional for the periodic waves of the Benjamin–Ono equation based on the conservation of momentum, energy, and a higher-order energy, which exists due to integrability. Positivity of the second variation of the Lyapunov functional for perturbations in Sobolev space $H^1(\mathbb{R})$ is established by utilizing the basis properties of eigenfunctions of the linearized Benjamin–Ono equation in $L^2(\mathbb{R})$. Coercivity of the Lyapunov functional for co-periodic and multiple-periodic perturbations yields the nonlinear orbital stability of the periodic waves in the periodic domain.

ALAN RIQUIER, University of Toronto
A Hamiltonian Description of Overturning Water Waves

Extending the ideas of Craig (2017), we propose a geometric generalization of the Water Waves equations (the Zakharov-Craig-Sulem formulation of Euler's free-surface equations) in physical dimension 2 or 3 where the fluid's free surface is represented by a parametrized curve or surface instead of the graph of some function, thus providing a new model for possibly-overturning, inviscid and irrotational oceanic surface waves. Advection of the free surface is achieved in a Lagrangian manner.

We also introduce a novel variational principle with prescribed tangential velocity for breaking waves, along with the corresponding complete Hamiltonian structure describing both normal and tangential movements of the free surface.

JAMES ROWAN, UT Austin

CATHERINE SULEM, University of Toronto

THIERRYLAURENS, University of Michigan
Rogue waves of infinite order

Rogue waves of infinite order are a family of special solutions to the focusing cubic nonlinear Schrödinger and modified Korteweg–de Vries equations on the line. They are not explicit, but each spatial profile is smooth and belongs to the space L^2. Remarkably, these solutions do not resolve into a superposition of solitons and radiation, as can be seen from their slow $t^{-1/3}$ temporal decay. Indeed, as these profiles are not in L^1, they lie outside the scope of known soliton resolution results, and are not amenable to classical inverse-scattering methods.

In this talk, we will discuss a new approach to the scattering theory for these infinite-order rogue waves. This is based on joint work with Deniz Bilman and Peter D. Miller.

KATJA VASSILEV, University of Chicago
Derivations of one-dimensional kinetic equations

Wave kinetic equations give an effective description of dispersive systems. In this talk, we look at various 1D dispersive systems and show how a diagrammatic expansion of the solution to the dispersive system gives a framework for these derivations. In

particular, we discuss the features of these dispersive systems which restrict the timescale to which these derivations can currently be done. Such features include the dimension of the system, resonances in the nonlinearity, and lack of invariances.

PENG YAN, Concordia University

Dark soliton gases in the defocusing nonlinear Schrödinger hydrodynamics

The defocusing nonlinear Schrödinger equation admits dark solitons under finite density boundary conditions. In this work, we construct dark soliton gas potentials of arbitrary genus by considering the limit of $\mathcal{N}$-dark soliton solutions as $\mathcal{N} \to \infty$. We establish a Fredholm determinant representation for the limiting potential and an associated Riemann–Hilbert problem. We then rigorously investigate the large-space and long-time asymptotic behaviours using the Deift–Zhou nonlinear steepest descent method. In the large-space limit, the genus-N dark soliton gas potential approaches a genus-N finite-gap solution as $x \to -\infty$ and the background 1 as $x \to +\infty$. In the long-time evolution, as the self-similar variable $\xi = x/t$ increases, the solution exhibits a cascade of asymptotic regimes, passing through unmodulated and modulated genus-N regions, progressively reducing the genus down to the planar region (unmodulated genus-0 region). This is joint work with Marco Bertola, Deng-shan Wang and Dinghao Zhu.