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Dark soliton gases in the defocusing nonlinear Schrödinger hydrodynamics

The defocusing nonlinear Schrödinger equation admits dark solitons under finite density boundary conditions. In this work, we construct dark soliton gas potentials of arbitrary genus by considering the limit of $\mathcal{N}$-dark soliton solutions as $\mathcal{N} \to \infty$. We establish a Fredholm determinant representation for the limiting potential and an associated Riemann–Hilbert problem. We then rigorously investigate the large-space and long-time asymptotic behaviours using the Deift–Zhou nonlinear steepest descent method. In the large-space limit, the genus-N dark soliton gas potential approaches a genus-N finite-gap solution as $x \to -\infty$ and the background 1 as $x \to +\infty$. In the long-time evolution, as the self-similar variable $\xi = x/t$ increases, the solution exhibits a cascade of asymptotic regimes, passing through unmodulated and modulated genus-N regions, progressively reducing the genus down to the planar region (unmodulated genus-0 region). This is joint work with Marco Bertola, Deng-shan Wang and Dinghao Zhu.