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Computer-Assisted Proofs of Localized Patterns in the Planar Swift-Hohenberg Equation

The Swift-Hohenberg equation is a classical model for pattern formation, exhibiting a rich variety of spatially periodic and localized structures. While numerical simulations suggest the existence of intricate localized patterns in two spatial dimensions, establishing their existence rigorously remains challenging. In this talk, I will present a computer-assisted approach for proving the existence of smooth, stationary, non-radially symmetric localized solutions of the planar Swift-Hohenberg equation. Our method combines a Newton-Kantorovich argument with Fourier analysis, analytic estimates, and rigorous numerical computations to validate solutions on the unbounded plane. We also establish that these localized patterns arise as limits of families of spatially periodic solutions as their periods tend to infinity. This is joint work with Matthieu Cadiot (École Polytechnique, France) and Jean-Christophe Nave (McGill).