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Mertens' Bias

The Möbius function $\mu(n)$ gives information about the number of prime factors in squarefree numbers, since it returns $+1$ (or -1) if the input has an even (or odd) number of prime factors. Its summatory function: Mertens' function $M(x) = \sum_{n \leq x} \mu(n)$ thus measures a type of 'prime number race': which of these two kinds of squarefree numbers are more abundant up to x.

In this talk, we discuss Mertens' conjecture : $|M(x)| < \sqrt{x}$ for all $x > 1$. This conjecture has been proven false in 1985, but a counterexample has yet to be found, although the first counterexample is believed to be some $10^{16} < x < 10^{10^{18}}$. Leveraging probabilistic methods from the study of prime number races, we compute the logarithmic density of the set of counterexamples to Mertens' conjecture, i.e., how often the conjecture fails.