Nonlinear and Geometric PDEs
EDP non-linéaires et géométriques
(Org: **Joshua Flynn** (Massachusetts Institute of Technology) and/et **Jérôme Vétois** (McGill University))

LORENZO CARLETTI, University of Oxford
Optimal constants for critical Sobolev Inequalities of high order on manifolds.

Let (M, g) be a closed Riemannian manifold of dimension n, and $k \geq 1$ be an integer such that $n > 2k$. We study the optimal constant problem for the Sobolev inequality coming from the embedding of the space $H^k(M)$ into $L^{2^\sharp}(M)$, where $2^\sharp = \frac{2n}{n-2k}$ is the critical Sobolev exponent. We investigate whether the first optimal constant is attained with the same value as in the Euclidean setting. We will show that the geometry of the manifold plays a crucial role in the validity of the optimal inequality, depending on the nature of the lower-order remainder term that is chosen. We follow a strategy of proof that works independently of the order of the problem, which relies on the study of the precise pointwise behavior of an asymptotically singular sequence of solutions to critical polyharmonic equation with diverging coefficients. Part of this talk is based on joint work with Frédéric Robert from IECL Nancy.

JEFFREY CASE, The Pennsylvania State University
Sharp Michael-Simon inequalities and the extrinsic Yamabe constant

The extrinsic Yamabe constant is a conformal invariant of submanifolds that computes the best constant in the L^2-Michael–Simon inequality. In this talk I will describe its basic properties, explain how to compute it on conformal submanifolds of the sphere, and apply it to the nonexistence of compact minimal submanifolds and obstructions to regularity in the σ_k-Loewner–Nirenberg problem. This talk is based on joint works with Dawit Mengesha, Samuel Perez-Ayala, and Aaron J. Tyrrell.

NGUYEN LAM, Memorial University of Newfoundland
Doubly sharp Poincaré–Sobolev inequalities on hyperbolic spaces

We study when one can subtract the full spectral gap $\left(\frac{N-1}{p}\right)^p$ of the p-Laplacian on the hyperbolic space $\mathbb{H}^N$ and still keep the sharp Euclidean Sobolev constant. Using a change of function which keeps the L^p norm, together with a one-dimensional tangent line inequality, we establish such doubly sharp inequalities with explicit remainder terms in new ranges of p, including $N = 3$ and $2 \leq p < 3$. The same method also gives sharp logarithmic Sobolev inequalities with the full spectral gap. This is joint work with A. X. Do, D. Ganguly and G. Lu.

GALEN NOVELLO, University of New Mexico
Sharp Decay of Solutions to p-Fractional equations with Hardy Potentials

In this talk I will present a sharp decay for non-negative solutions of p-fractional equations with Hardy potentials with a critical exponent in the setting of homogeneous groups. In particular the decay of solutions to $\mathcal{L}_{s,p}u = \frac{|u|^{p^*_\alpha(s)-2}u}{|g|^\alpha}$ where $\mathcal{L}_{s,p}$ is the fractional p-Laplacian and $p^*_\alpha(s)$ is the fractional Hardy-Sobolev exponent will be discussed.

RAQUEL PERALES, Centro de Investigación en Mátematicas, Guanaujato

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JACOB REZNIKOV, Massachusetts Institute of Technology
A sharp Hessian estimate for Green's functions in positive curvature

On manifolds with nonnegative Ricci curvature, Colding proved a sharp gradient estimate and monotonicity formulas for the Green's function, and Cosmin Manea extended these to Ricci curvature bounded below by $(n-1)k > 0$, using the Green's function G of $-\Delta + n(n-2)k/4$, which on the round sphere is the conformal Laplacian. I will present a sharp Hessian inequality for this Green's function on closed Einstein manifolds with sectional curvature at least $k > 0$: setting $s = G^{1/(2-n)}$, we show that $\nabla^2 s^2 + k s^2 g \leq 2g$ away from the pole, with equality at even one nonzero vector only for the round sphere. This can be viewed as a Green's function analogue of the Hessian comparison theorem, and as an elliptic, positively curved counterpart to Hamilton's matrix Li–Yau inequality; it also extends an earlier estimate of Park from nonnegative curvature, in sharp form. As applications we obtain a two-parameter family of sharp monotone quantities and a comparison inequality $s_p^2 + s_q^2 \leq 4/k$ for the Green's functions of a suitable pair of points $p \neq q$, again rigid only for the round sphere. This is joint work with Cosmin Manea.

DIMITER VASSILEV, University of New Mexico

Regularity of Solutions to Non-Local Operators on Homogeneous Groups

We prove a Harnack inequality and show Hölder continuity of solutions to non-linear nonlocal operators on homogeneous groups.

SAMUEL ZEITLER, Carnegie Mellon University

Positivity and non-positivity results for the sixth-order Q-curvature of conformal metrics

Given $n, m \in \mathbb{N}$ such that $n \geq 2m \geq 4$, letting g be a conformally Euclidean metric on $\mathbb{R}^n$, we consider the question of positivity of the lower-order Q-curvatures $Q_g^{(2k)}$ for $k \in \{1, \dots m-1\}$ when $Q_g^{(2m)}$ is assumed to be nonnegative and not identically zero. We assume moreover that the scalar curvature of the metric g is nonnegative near infinity if $n = 2m$ or that $Q_g^{(2m)}$ satisfies a slow decay barrier condition near infinity if $n > 2m$. A positive answer to this question for the second and fourth order Q-curvatures was obtained by Li and Xu. Considering the case where $m \geq 4$ and $k = 3$, we obtain a positive result for this question when $n \in \{2m, 2m+1, \dots, 4m-6\}$. On the other hand, in surprising contrast with previous literature, we find that the answer to this question is negative when $k = 3$ and $n \geq N_m$ for some $N_m \in \mathbb{R}$. This talk is based on a joint work with Jérôme Vétois.