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Optimal constants for critical Sobolev Inequalities of high order on manifolds.

Let (M, g) be a closed Riemannian manifold of dimension n, and $k \geq 1$ be an integer such that $n > 2k$. We study the optimal constant problem for the Sobolev inequality coming from the embedding of the space $H^k(M)$ into $L^{2^\sharp}(M)$, where $2^\sharp = \frac{2n}{n-2k}$ is the critical Sobolev exponent. We investigate whether the first optimal constant is attained with the same value as in the Euclidean setting. We will show that the geometry of the manifold plays a crucial role in the validity of the optimal inequality, depending on the nature of the lower-order remainder term that is chosen. We follow a strategy of proof that works independently of the order of the problem, which relies on the study of the precise pointwise behavior of an asymptotically singular sequence of solutions to critical polyharmonic equation with diverging coefficients. Part of this talk is based on joint work with Frédéric Robert from IECL Nancy.