JACOB REZNIKOV, Massachusetts Institute of Technology

A sharp Hessian estimate for Green's functions in positive curvature

On manifolds with nonnegative Ricci curvature, Colding proved a sharp gradient estimate and monotonicity formulas for the Green's function, and Cosmin Manea extended these to Ricci curvature bounded below by $(n-1)k > 0$, using the Green's function G of $-\Delta + n(n-2)k/4$, which on the round sphere is the conformal Laplacian. I will present a sharp Hessian inequality for this Green's function on closed Einstein manifolds with sectional curvature at least $k > 0$: setting $s = G^{1/(2-n)}$, we show that $\nabla^2 s^2 + k s^2 g \leq 2g$ away from the pole, with equality at even one nonzero vector only for the round sphere. This can be viewed as a Green's function analogue of the Hessian comparison theorem, and as an elliptic, positively curved counterpart to Hamilton's matrix Li–Yau inequality; it also extends an earlier estimate of Park from nonnegative curvature, in sharp form. As applications we obtain a two-parameter family of sharp monotone quantities and a comparison inequality $s_p^2 + s_q^2 \leq 4/k$ for the Green's functions of a suitable pair of points $p \neq q$, again rigid only for the round sphere. This is joint work with Cosmin Manea.