Harmonic Analysis and Geometric Measure Theory
L'analyse harmonique et la théorie géométrique de la mesure
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MENGMENG DOU, University of British Columbia
Regularity of multi-parameter Fourier integral operators

We investigate a new class of Fourier integral operators $\mathbf{F}$ defined in $\mathbb{R}^{\mathbf{N}}$. Their symbols are allowed to satisfy a differential inequality with certain multi-parameter characteristic. We prove these operators of order $-\frac{\mathbf{N}-1}{2}$ bounded from the classical, atom decomposable H^1-Hardy space to $L^1(\mathbb{R}^{\mathbf{N}})$. As a result, we obtain a sharp L^p-estimate.

Simultaneously, a generalized Sobolev L^p-space is introduced. We establish the Sobolev L^p-norm inequality for convolutions with a distribution having singularity on the unit sphere. As an application, we give new a priori estimate for the solution of wave equations by requiring less regularity on the source term and initial data.

MARCO FRACCAROLI, University of Massachusetts Lowell
Power weight inequalities for spherical maximal functions

The average of a Schwartz function f over the sphere of radius t and centre $x \in \mathbb{R}^d$ is

$$A_t f(x) := \int_{\mathbb{S}^{d-1}} f(x + t\omega) d\sigma(\omega),$$

where σ is the normalized surface measure on the sphere $\mathbb{S}^{d-1}$. For any set of dilations $\mathcal{E} \subseteq (0, \infty)$, the spherical maximal function $M_{\mathcal{E}}$ is defined by

$$M_{\mathcal{E}} f(x) := \sup_{t \in \mathcal{E}} |A_t f(x)|.$$

When $\mathcal{E} = (0, \infty)$, Stein and Bourgain proved that $M_{\mathcal{E}}$ is bounded on $L^p(\mathbb{R}^d)$ if and only if $p > \frac{d}{d-1}$ in dimensions $d \geq 3$ and $d = 2$, respectively. In fact, for any $\mathcal{E}$, the lower bound on p depends on the Minkowski dimension of $\mathcal{E}$, as shown by Seeger, Wainger, and Wright.

Moreover, when $\mathcal{E} = (0, \infty)$, Duoandikoetxea and Vega established the weighted L^p estimates for $M_{\mathcal{E}}$ with power weights $w_\alpha(x) = |x|^\alpha$. Jointly with Roos and Seeger we identified the relevant dimensional parameter for the weighted L^p estimates for any $\mathcal{E}$, its Assouad spectrum, settling an open problem appearing in the work of Duoadikoetxea and Seijo.

ROBERT FRASER, Wichita State University

KAIYI HUANG, University of Wisconsin-Madison
Finner-like inequalities in the Heisenberg group

We completely characterize the range of L^p-boundedness of certain multilinear Radon-like transforms involving vertical projections in the Heisenberg group.

KAIWEN JIN, University of Wisconsin-Madison
Fourier restriction to hyperbolic rectangles and an application

In this talk, we study the Lebesgue space inequalities for extension operators associated with hyperbolic surfaces in $\mathbb{R}^3$ over rectangular regions. In the polynomial-method range, we characterize the corresponding operator norms in terms of the side-lengths. As an application, we present new restriction estimates for a class of hypersurfaces with additive structure.

JORIS ROOS, University of Massachusetts Lowell

HRIT ROY, University of Jyväskylä

IGNACIO URIARTE-TUERO, University of Toronto

Muckenhoout Ap weights, BMO, distance functions and Hardy Sobolev inequalities

Vasin (for $n = 1$) and Anderson, Lehrbäck, Mudarra, and Vähäkangas (for $n > 1$) provided a geometric characterization of the sets $E \subset \mathbb{R}^n$ so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_1 weight for some $\alpha > 0$. We provide a geometric characterization of the sets $E \subset \mathbb{R}^n$ (which we call median porous sets) so that $w = dist(\cdot, E)^{-\alpha}$ is a Muckenhoupt A_p weight for some $\alpha > 0$ (given any $1 < p \leq \infty$).

Given $1 < p \leq \infty$, we also find the precise range of exponents α so that $w = dist(\cdot, E)^{-\alpha} \in A_p$ (in analogy to the $p = 1$ case done by Anderson, Lehrbäck, Mudarra, and Vähäkangas).

With our characterization we prove that $\mathbb{R}^n \setminus E$ supports a Hardy-Sobolev inequality if E is an appropriate median porous set. All previous such results that we are aware of make the strictly stronger assumption that the set E is porous. The proofs rely on a new median-value characterization of BMO. Joint work with Marcus Pasquariello.

Time permitting, I will also explain the one-sided version of these theorems on the real line, joint work with Alptekin Can Goksan.