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Power weight inequalities for spherical maximal functions

The average of a Schwartz function f over the sphere of radius t and centre $x \in \mathbb{R}^d$ is

$$A_t f(x) := \int_{\mathbb{S}^{d-1}} f(x + t\omega) d\sigma(\omega),$$

where σ is the normalized surface measure on the sphere $\mathbb{S}^{d-1}$. For any set of dilations $\mathcal{E} \subseteq (0, \infty)$, the spherical maximal function $M_{\mathcal{E}}$ is defined by

$$M_{\mathcal{E}} f(x) := \sup_{t \in \mathcal{E}} |A_t f(x)|.$$

When $\mathcal{E} = (0, \infty)$, Stein and Bourgain proved that $M_{\mathcal{E}}$ is bounded on $L^p(\mathbb{R}^d)$ if and only if $p > \frac{d}{d-1}$ in dimensions $d \geq 3$ and $d = 2$, respectively. In fact, for any $\mathcal{E}$, the lower bound on p depends on the Minkowski dimension of $\mathcal{E}$, as shown by Seeger, Wainger, and Wright.

Moreover, when $\mathcal{E} = (0, \infty)$, Duoandikoetxea and Vega established the weighted L^p estimates for $M_{\mathcal{E}}$ with power weights $w_\alpha(x) = |x|^\alpha$. Jointly with Roos and Seeger we identified the relevant dimensional parameter for the weighted L^p estimates for any $\mathcal{E}$, its Assouad spectrum, settling an open problem appearing in the work of Duoadikoetxea and Seijo.