Hypercomplex Analysis: Function Theory, Operator Theory, and Applications
L'analyse hypercomplexe : la théorie des fonctions, la théorie des opérateurs, et leurs applications
(Org: **Pierre-Olivier Parisé** (Université du Québec à Trois-Rivières))

DANIEL ALPAY, Chapman University

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FABRIZIO COLOMBO, Politecnico di Milano
The nonlocal Fourier law and the fractional heat equation

In this talk, we discuss fractional powers of vector operators, focusing on the gradient operator with nonconstant coefficients, which turns out to be bisectorial. Our approach is based on the spectral theory on the S-spectrum. The main difficulty is that, unlike in the case of sectorial operators, the spectrum of a bisectorial operator extends to the negative real axis, where the fractional power function is not analytic. To overcome this, we introduce a new definition of fractional powers adapted to the bisectorial setting. Building on previous results for bisectorial vector operators and weak solutions, we extend this definition to abstract vector operators. Finally, we apply it to the gradient operator and show that its fractional powers provide a rigorous foundation for nonlocal Fourier laws in heat propagation.

MARCO ANTONIO PÉREZ DE LA ROSA, Universidad de las Américas Puebla
A Mathematical Framework for Two-Dimensional Diffraction Phenomena Based on Biquaternions

This talk presents a mathematical framework for describing two-dimensional diffraction phenomena using biquaternions. In particular, biquaternionic counterparts of the two-dimensional Helmholtz–Kirchhoff integral theorem, the Fresnel–Kirchhoff diffraction formula, and the Rayleigh–Sommerfeld diffraction formula are derived. These results provide the foundation for a biquaternionic formulation of two-dimensional diffraction phenomena capable of treating scalar and vector fields within a unified framework. This approach offers an attractive alternative to traditional methods, particularly in settings where simultaneous analysis of scalar and vector components is required. As a special case, the classical one-dimensional Fresnel–Kirchhoff diffraction formula is recovered. This result serves as a consistency check and confirms the mathematical validity of the proposed framework. This is a joint work with Wanderson Maia Pimenta.

DAVID EELBODE, University of Antwerp
Multi-complex numbers and spinors

Spinors are crucial and ubiquitous in both mathematics and physics, but they are less intuitive than rotations - despite the fact that spinors "merely" define an irreducible representation for the rotation (or spin) group. In this lecture we will try to explain where spinors really come from, and what they are trying to express geometrically. To do that, we will start from the Cartan-Dieudonné theorem (which factorises a rotation in terms of reflections) in the setting of geometric algebras (also known as Clifford algebras). Using simple multiplication rules in this algebra, which we will also derive from the geometry itself, we

will then see how spinors spontaneously emerge as certain multivectors in this algebra. We will then proceed with noting that there are pure spinors, which have a nice and geometric interpretation, but in dimensions $m \geq 8$ one gets impure spinors as well. These are very interesting, leading to for instance models in the geometric algebra for exotic number systems such as the octonions and sedenions, but less intuitive. We will explain the connection with multi-complex numbers, and if time permits also show how these exotic structures are embedded.

JOSÉ OSCAR GONZÁLEZ-CERVANTES, Instituto Politécnico Nacional, ESFM

PIERRE-OLIVIER PARISÉ, Université du Québec à Trois-Rivières
Hypercomplex fractals obtained by conjugations

In 2000, D. Rochon used the bicomplex numbers to define a four-dimensional generalization of the classical Mandelbrot set. These numbers were later used to generalize the Multibrot sets to higher dimensions.

In this talk, by adding a conjugate to the iteration that generates the bicomplex Mandelbrot set, I will present a four-dimensional generalization of the Multicorn sets introduced by Dierk Schleicher. I will also establish a rigorous lower bound on the distance from a point to the bicomplex Multicorns, and use it in a ray-marching algorithm to visualize their three-dimensional cross-sections.

This is joint work with Dominic Rochon.

IRENE MARIA SABADINI, Politecnico di Milano
Spinor Dirac operator on manifolds with boundary conditions and S-resolvent estimates

In this talk, we present recent results on the spectral theory based on the S-spectrum applied to the Dirac operator D on manifolds subject to homogeneous Dirichlet and Robin-type boundary conditions. Applying this spectral theory requires considering D^2, which is expressed as the sum of a second-order Laplacian and a curvature operator, in accordance with the Schrödinger–Bochner–Weitzenböck theorem. This makes the analysis substantially different from existing approaches in the literature. Despite this, we are still able to obtain bisectorial-type estimates for the S-resolvent.