Geometric Problems in PDEs
Problèmes géométriques pour les EDP
(Org: **Lia Bronsard** and/et **Kenneth DeMason** (McMaster University))

ALMUT BURCHARD, University of Toronto
A new characterization of the 2-sphere

I will present an elementary proof of Hersch's inequality and Cheng's theorem, both characterizing the round metric on S^2. A related question about spatial monotonicity of heat kernels provides a third characterization. (Joint work with Ángel D. Martínez)

KENNETH DEMASON, McMaster University
A unified approach to ABP and optimal transport proofs of isoperimetric inequalities

Of the many proofs of the isoperimetric inequality, the two based on the Alexandrov-Bakelman-Pucci technique and optimal transportation share remarkably similar features. We connect these approaches through a one-parameter family of optimal transportation problems that interpolates between the ABP and Brenier potentials. Each solution gives an elementary proof of the isoperimetric inequality. The method extends to weighted anisotropic isoperimetric inequalities in convex cones, yielding the Wulff and Lions-Pacella inequalities as in [Cabré–Ros-Oton–Serra '16]. In the classical case, we show the two potentials coincide precisely when the domain is a ball.

NICK EDELEN, University of Notre Dame
Entire area-minimizing surfaces in $\mathbb{R}^n$

It's well-known that any holomorphic subvariety is area-minimizing in its homology class. In this talk we consider the converse question, and we show that any entire area-minimizing 2-surface $M^2 \subset \mathbb{R}^n$ of density two at infinity (that is, asymptotic to a pair of planes, possibly coincident) is either planar or is a degree-2 algebraic variety in some $\mathbb{C}^2 \subset \mathbb{R}^n$. We additionally prove algebraicity of M^2 when the tangent cone at infinity is multiplicity-one. This is joint work with Luis Atzin Franco Reyna and Paul Minter.

AILANA FRASER, University of British Columbia
Free boundary minimal surfaces in rectangular prisms

In this talk we will describe recent work using eigenvalue optimization to construct free boundary minimal surfaces in products of Euclidean balls. We will briefly outline the general theory and apply it in a specific case to explain and generalize the Schwarz P-surface, which is a free boundary minimal surface in the three dimensional cube with one boundary component on each face of the cube. We will show how the method can be used to construct such surfaces in rectangular prisms with arbitrary side lengths. We will also show that when the side lengths are not all the same, there are at least two minimal surfaces of the type of the Schwarz P-surface and both are embedded.

COLIN GRANT, McMaster University
Jacobi fields of minimizing planar triple bubbles

A classic question in geometry is as follows: given k prescribed volumes in $\mathbb{R}^n$, what is the least perimeter/surface area configuration in which these volumes can be arranged? The standard double bubble is the globally minimizing configuration for two volumes; it was recently shown that all Jacobi fields of the standard double bubble are generated by infinitesimal rotations

and translations (Di Matteo, 2019). We extend this work to the globally minimizing planar standard triple bubble, and consider the planar 'caterpillar' triple bubble. This work was supervised by Dr. Lia Bronsard and Dr. Ken DeMason.

PENGFEI GUAN, McGill University
An inverse-curvature-type hypersurface flow in space forms

We study an inverse-curvature-flow in hyperbolic space and the hemisphere. We introduce explicit initial radial-slope conditions under which smooth admissible star-shaped hypersurfaces evolve for all time and converge exponentially to a geodesic sphere. These first-order conditions are compatible with nonconvex admissible hypersurfaces and are implied by the corresponding horo-convexity assumptions for radial graphs. The monotonicity of quermassintegrals along the flow yields sharp geometric inequalities, with equality precisely for geodesic spheres. This is a joint work with Junfang Li.

FANG HONG, McGill University
Santaló's Problem and Minkowski-Type Inequalities in Cartan-Hadamard Three-Manifolds

The classical Minkowski inequality states that spheres minimize total mean curvature among convex surfaces of prescribed area in Euclidean three-space. In hyperbolic space, the corresponding problem, posed by L. A. Santaló, exhibits a richer geometry. In this talk, we will survey Minkowski-type inequalities in Cartan–Hadamard three-manifolds and present a refinement of Ghomi–Spruck's inequality. We will then discuss Santaló's minimization problem and recent results on the geometry of its minimizers.

LUANA JOST, Institut de Mathématiques de Toulouse
An a priori estimate for the energy growth of minimizing entire solutions of the Ginzburg-Landau equation

In higher dimensions, the classification of minimizing entire solutions of the Ginzburg-Landau equation remains an open problem. With a view toward this classification, we study energy estimates of such minimizing solutions on large balls B_R in $\mathbb{R}^n$, $n \geq 2$. This is joint work with Étienne Sandier.

SIYUAN LU, McMaster University
Recent progress on Hessian quotient equations

Hessian quotient equations are closely related to convex geometry, complex geometry and special Lagrangian geometry. In this talk, I will discuss some recent progress on Hessian quotient equations from analytic point of view, including rigidity of entire solutions and a priori estimates.

GUANYING PENG, Worcester Polytechnic Institute
Rigidity of Aviles-Giga limit states with L^p entropy productions

The Aviles-Giga functional is a classical energy functional related to the theories of smectic liquid crystals and thin-film blisters. Its variational analysis has received significant attention. The Aviles-Giga energy provides sequences of maps converging to weak solutions m of the 2D Eikonal equation, whose entropy productions are Radon measures controlled by the energy. It is conjectured that these entropy production measures are concentrated on the one-dimensional jump set of m. In particular, the entropy production measures should vanish if they coincide with L^p functions. In recent work, we establish this consequence of the conjecture under natural boundary conditions and the assumption that p is not too small. This is joint work with Xavier Lamy and Andrew Lorent.

RÉMY RODIAC, Université Côte d'Azur
Stability conditions for the Ginzburg-Landau equations in the mean-field regime

We will discuss second order stability conditions for limits of stable solutions to the Ginzburg-Landau equations (with and without magnetic field) in the mean-field regime. This is a regime where the number of vortices diverges (typically their number is of order $|\log \varepsilon|$). The conditions we find are obtained by passing to the limit in the second inner variations of the Ginzburg-Landau energy.

REGINA ROTMAN, University of Toronto

CARLOS VALERO SAEZ, ICMAT
The Calderón problem for harmonic spinors

We consider the Calderón problem for the Dirac operator over locally conformally real-analytic (LCRA) manifolds with boundary. More precisely, we define an analogue of the Dirichlet-to-Neumann map for the Dirac operator, and show that if the metric is locally conformal to a real-analytic metric in some coordinates, then this map determines the conformal class of the manifold. The class of LCRA manifolds includes conformally Einstein manifolds, as well as all smooth surfaces.

CARLOS VALERO SAEZ, Instituto de Ciencias Matemáticas

ALINA STANCU, Concordia University
On the Gaussian Torsion

The Ornstein–Uhlenbeck operator is the natural analogue of the Laplacian on $\mathbb{R}^n$ equipped with the standard Gaussian measure. Although it is not translation invariant, it shares many features with the Laplace operator. In this talk, we consider the torsion function and torsional rigidity associated with the Ornstein–Uhlenbeck operator. In the classical Euclidean setting, torsional rigidity satisfies a Brunn–Minkowski inequality on convex domains. Moreover, the square root of the torsion function is concave on every convex domain, while the torsion function itself is concave under suitable smoothness and curvature assumptions on the boundary. In joint work with Xuan Hien Nguyen, we show that the Gaussian setting is strikingly different. The natural Brunn–Minkowski-type inequality conjectured for Gaussian torsional rigidity fails in every dimension greater or equal two, even for centrally symmetric, smooth, strictly convex domains. We also construct, in every such dimension, centrally symmetric, uniformly convex smooth domains whose Ornstein–Uhlenbeck torsion function is not concave.

DOMINIK STANTEJSKY, Université de Lorraine
Minimizing isoperimetric partitions in higher dimensions via variational autoencoders

In this talk, I will present a numerical algorithm for an isoperimetric partitioning problem with volume constraints. The algorithm is based on a parametrization of each volume as image of reference volumes under diffeomorphisms. These parametrizations are represented via variational autoencoder neural networks that are trained together to minimize total perimeter of their boundaries while respecting volume and partitioning constraints. For the computations we rely on Monte-Carlo methods, thus allowing us to apply our algorithm in high dimensions.

HUNTER STUFFLEBEAM, Johns Hopkins University
Width Stability of Three-Manifolds

In the 1990's, researchers such as Colding and Petersen investigated the stability of invariants such as the radius, volume, and Laplace eigenvalues λ_k of closed Riemannian manifolds with $\mathrm{Ric}_g \geqslant (n-1)g$. They showed that almost maximality of either

the radius or volume, or minimality of λ_{n+1} (listed with multiplicity), is equivalent to the closeness of the manifold of interest to the unit round sphere.

We contribute another invariant to this list in dimension $n = 3$, by studying the almost-rigid properties of the first Almgren-Pitts width $\omega_1(M^3, g)$ as a non-linear, minimal-surface-based analogue of Petersen's work with the Laplace spectrum. Our main techniques involve the Ricci flow and the convergence/compactness theory of minimal surfaces.

IHSAN TOPALOGLU, Virginia Commonwealth University

Minimality and stability of the ball in the liquid drop model with screened interactions

I will present on the volume-constrained minimization of a functional given the sum of the perimeter functional and a nonlocal repulsive term defined through the Yukawa kernel. In this functional the competition between surface tension and repulsion is governed by two independent quantities: the screening rate and the volume. For this problem, above an explicit critical screening rate, minimizers exist at every volume, and below an explicit volume threshold minimizers exist for every screening rate. Moreover at small volume the ball is the unique minimizer up to translation, uniformly in the screening rate. Finally, there exists a sharp volume threshold, depending on the screening rate, for the ball to be a stable volume-constrained critical point. This is a joint work with Lia Bronsard and Kenneth DeMason.

CHONG WANG, Washington and Lee University

Recent Works on Variational Problems

For work 1, the functional of the variational problem consists of two terms: the perimeter of the input subset and an interaction integral with a kernel that is the sum of a logarithmic function and a quadratic function. This kernel is bounded below and tends to infinity at zero and infinity. A single disk is always a critical point of the functional, but its stability depends on the parameters of the problem. When the parameters are in a suitable range there exist assemblies of multiple perturbed disks that are stable critical points. For work 2, the functional of the variational problem combines an interface short-range interaction energy promoting micro-domain growth with a Coulomb-type long-range interaction energy which prevents micro-domains from unlimited spreading. Here we consider a scenario in which two species are dominant, and one species is vanishingly small. In this scenario two energy levels are distinguished: the zeroth-order energy encodes information on the optimal arrangement of the dominant constituents, while the first-order energy gives the shape of the vanishing constituent. This first-order energy also shows that, for any optimal configuration, the vanishing phase must lie on the boundary between the two dominant constituents and form lens clusters.

SHUOYANG WANG, McMaster University

Stationary States on Periodic Metric Graphs with Nonzero Boundary Conditions

Stationary states of the nonlinear Schödinger (NLS) equation are considered on periodic metric graphs. We study stationary states with nonzero boundary conditions which model black solitons in optics. For a general graph with a single edge connecting compact cores symmetrically and periodically, we prove the existence of an energy minimizer pinned to the point of the edge symmetry. For the neckless graph example, we prove the coexistence of two energy minimizers pinned to two points of symmetry: one is in the middle of the link connected to two loops and the other one is in the middle of the loop connected to two links. In the semi-classical limit, which is equivalent to the limit of long edges, we prove that no other stationary states exist, whereas the link- pinned state is a local minimizer of energy and the loop-pinned state is a saddle point of energy without the pinning condition. We also prove that both are unstable in the time evolution of the NLS equation in such a way that the energy minimizer is strongly unstable with a real eigenvalue and the saddle point of energy is weakly unstable with a quadruplet of complex eigenvalues. This is the joint work with Dr. Stefan Le Coz (Toulouse) and Dr. Dmitry E. Pelinovsky (McMaster).