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Recognizing convergence actions yields several questions

An action of a group G on topological space M is said to be a convergence action if the induced action on the space of distinct triples of M is properly discontinuous. In joint work with Ding, we show that every convergence action of a group G on a perfect compactum (a compact metrizable space with no isolated points) is realized as the boundary action of a proper isometric action of G on a hyperbolic space X. This yields intriguing consequences and questions. Taking G to be trivial realizes every perfect compactum as a Gromov boundary. Applying the theorem to the one-point compactification of a Cayley graph gives every finitely generated group a proper action on a hyperbolic space with that boundary. Given a hyperbolic group G, every subgroup H of G acts as a convergence group on ∂G, and hence admits a proper isometric action on a hyperbolic space X_H with ∂G as its Gromov boundary. How do the spaces X_H compare as H varies, given that they all have the same boundary ∂G? A closed hyperbolic surface group admits convergence actions on S^2 with limit set a circle (quasi-Fuchsian) or the whole sphere (Cannon–Thurston). How do the resulting hyperbolic spaces compare?