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On $H = W$ in Banach Function Spaces

In this talk, I will present some of our recent work—conducted in collaboration with D. Cruz-Uribe (UA, USA) and S. Rodney (CBU, Canada)—on what is known in the literature as the $H = W$ problem. We consider a Banach function space $X(\Omega)$ and its corresponding Sobolev space $W^1_X(\Omega)$.

Our central question is when is the space $W^1_X(\Omega)$ equal to the space $H^1_X(\Omega)$. Here, $H^1_X(\Omega)$ is defined as the closure of the set $\mathrm{Lip}(\Omega) \cap W^1_X(\Omega)$ with respect to the norm $\|f\|_{W^1_X(\Omega)} = \|f\|_{X(\Omega)} + \|\nabla f\|_{X(\Omega)}$.

I will discuss the hypotheses we impose on $X(\Omega)$ that allow this equality to hold. Specifically, we assume the Fatou property, the absolute continuity of the norm, and a strong Muckenhoupt condition. I will state our main result under these assumptions and briefly explain why they are essential to the proof.

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