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Sharp regularity of maximal functions

The classical Hardy-Littlewood maximal function theorem states that the Hardy-Littlewood maximal operator is a bounded operator on $L^p(\mathbb{R}^d)$ if and only if $1 < p \leq \infty$. The main ingredient to the proof is the Vitali covering lemma, which roughly states that any bounded set of balls has a pairwise disjoint subset which represents the volume of the original set well.

In 1997 Juha Kinnunen proved the corresponding result for the gradient of the maximal function, i.e. that the $L^p(\mathbb{R}^d)$-norm of the gradient of the maximal function is controlled by the $L^p(\mathbb{R}^d)$-norm of the gradient of the function if $1 < p \leq \infty$. However, he provides no counterexample in the endpoint $p = 1$, and so in 2004 Hajłasz and Onninen formally posed the question if the endpoint gradient bound also holds.

We present current research on the problem, emphasizing the one dimensional case $d = 1$.