Eigenvalues and graphs
Les valeurs propres et les graphes
(Org: **Karen Meagher** and/et **Hermie Monterde** (University of Regina))

LUDOVICK BOUTHAT, University of Regina & University of Delaware
Entrywise Powers and Critical Exponents of Graphs

Sparse covariance and correlation matrices motivate the study of entrywise transformations that preserve both positive semidefiniteness and a prescribed pattern of zeros. The Schur product theorem guarantees that positive integer entrywise powers preserve positive semidefiniteness, while a classical theorem of FitzGerald and Horn identifies a sharp threshold for real powers in each dimension. Prescribing a pattern of zeros leads to a finer problem in which the structure of a graph enters the picture.

The critical exponent of a graph is the threshold beyond which every entrywise power preserves positive semidefiniteness for matrices with nonnegative entries and zeros at the nonedges. Understanding this invariant involves determining its possible values and identifying the graph properties that govern it. This talk will introduce some classical results and discuss recent surprising progress on these questions. Small graphs and structured families will serve as concrete examples throughout, highlighting the interplay between graph structure and matrix positivity.

STEVE BUTLER, Iowa State University
Gluing via block similarity matrices for distance matrices

A common method to form cospectral graphs is to glue two graphs on a common vertex. The justification is often carried out by the use of combinatorial interpretations of coefficients of characteristic polynomials that can be readily computed using sparseness of the related matrices. However, if the matrix is dense, e.g. a distance matrix, this approach does not work.

In this talk we will look at how to use block similarity matrices to form cospectral constructions via gluing which work for many classes of matrices, including the adjacency, Laplacian, signless Laplacian, distance, and generalized distance matrix.

This work is the culmination of two REU projects that happened at Iowa State University in recent years.

HOMER DE VERA, University of Manitoba

JAIME GARZA, University of Ottawa
Eigenvalues of Random Permutation Matrices

Random permutation matrices can be viewed as adjacency matrices of directed graphs consisting of disjoint cycles. Their eigenvalues are determined by the cycle structure of the underlying permutations, allowing spectral questions to be approached through the theory of random permutations. In this talk, we consider permutation matrices generated by the two-parameter Ewens–Pitman model. This model admits a sequential construction through the two-parameter Chinese restaurant process and includes uniform and Ewens permutations as special cases. We present joint work with Yizao Wang on the asymptotic behavior of cycle counts and linear eigenvalue statistics, highlighting the different fluctuation behavior that emerges beyond the classical Ewens setting.

CHRIS GODSIL, university of waterloo
Quantum and fractional automorphisms of graphs

We consider two generalizations of graph automorphisms.

A *quantum permutation* of index d is an $n \times n$ matrix whose entries are $d \times d$ projections, such that each row sums to I_d. (A quantum permutation of index one is an ordinary permutation.) A quantum permutation $\mathcal{P}$ of index d is a *quantum automorphism* of the graph X if $\mathcal{P}$ and $A(X) \otimes I_d$ commute.

We consider two generalizations of graph automorphisms.

A *quantum permutation* of index d is an $n \times n$ matrix whose entries are $d \times d$ projections, such that each row sums to I_d. (A quantum permutation of index one is an ordinary permutation.) A quantum permutation $\mathcal{P}$ of index d is a *quantum automorphism* of the graph X if $\mathcal{P}$ and $A(X) \otimes I_d$ commute.

A *fractional automorphism* of a graph is a doubly stochastic matrix S such that $AS = SA$. Any permutation matrix that commutes with $A(X)$ is a fractional automorphism.

These two extensions are related. If $\mathcal{P}$ is a quantum automorphism of X with index d, then for each $d \times d$ positive semidefinite matrix D with trace one, we can derive a fractional automorphism of X.

In my talk I will explain some of the relations between these extensions, and some of the reasons they are interesting.

KRYSTAL GUO, University of Amsterdam

Perfect state transfer, Cayley presentations, and quantum hardware

We study perfect state transfer (PST) on Cayley graphs. Bridging the classical theory of isomorphic Cayley graphs with quantum walks, we show that under mild hypotheses, Cayley graphs of groups with an abelian subgroup of index two, or of extraspecial p-groups with conjugacy-closed connection sets, are also Cayley graphs of abelian groups. Consequently, many known constructions of PST on dihedral, dicyclic, and extraspecial 2-groups are actually non-abelian presentations of abelian Cayley graphs. Conversely, we prove that for odd prime powers $q \geq 5$, the $\mathrm{SL}(2, q)$ graphs of Pantangi and Sin admit no abelian Cayley presentation, giving the first provably non-abelian infinite family of Cayley graphs with PST. We also construct an infinite family of Cayley graphs with peak state transfer.

Finally, I will briefly discuss experiments implementing continuous-time quantum walks with perfect and peak state transfer on a 156-qubit IBM Heron quantum processor.

Based on joint works with Arnbjörg Soffía Árnadóttir (arXiv:2608.20476) and Priscilla Awuah (arXiv:2610.20855).

MAX LEVIT, York University

Orthogonal signed graphs of degrees five and six

A signed graph is orthogonal if the distinct rows of its signed adjacency matrix are pairwise orthogonal. Examples include the signed K_6 associated with the antipodal quotient of the icosahedron, several infinite families of tessellations of the torus, and the two-eigenvalue signed hypercubes used in the resolution of the Sensitivity Conjecture.

In fact, all orthogonal signed graphs must have exactly two distinct (signed adjacency) eigenvalues, so we are dealing with a special case of the two-eigenvalue inverse eigenvalue problem for graphs.

I will discuss the examples and share our recent classification of the 5-regular orthogonal signed graphs. Time permitting, I will discuss partial results in the 6-regular case as well.

Based on joint work with Bojan Mohar and Behruz Tayfeh-Rezaie.

KAREN MEAGHER, University of Regina

Applications of eigenvalues of graphs for Erdos-Ko-Rado type results

My research is centred on results related to the famous Erdős-Ko-Rado (EKR) Theorem. My favourite proof method for this theorem is to define a graph with the property that intersecting sets are exactly the independent sets/cocliques in the graph and to use algebraic methods to understand the structure of this graph. This graph is typically called a derangement graph. It is often very symmetric, so tools from linear algebra and algebraic graph theory can be surprisingly effective. For example, the highly structured nature of theses derangement graphs allow us to determine its eigenvalues, which can be used to prove

the different EKR theorems. A family of variations of the EKR theorems are known as stability results, these demonstrate in different ways how robust version of EKR theorem can be. One type of stability result is to ask at what probability can edges can be randomly removed from some derangement graph without creating a subgraph that has, with a high probability, larger cocliques. The maximum value of this probability could be seen as measure of the robustness of the EKR theorem. I will show one way this probability can be determined using eigenvalues.

HERMIE MONTERDE, University of Regina
Bipartiteness, planarity, and vertex sedentariness

A vertex in a quantum walk on a graph is said to be *sedentary* if the quantum state assigned on that vertex tends to stay at home. Sedentary vertices are thought to be useful in the development of quantum memory for quantum computers. In this talk, we discuss recent results on the theory of vertex sedentariness using elementary graph-, matrix- and number-theoretic tools. In particular, we highlight rarity-type results in bipartite and planar graphs. If time permits, then we also discuss vertex sedentariness in integral graphs. Most of this talk is derived from the paper entitled *Sedentary quantum walks on bipartite and planar graphs* (arxiv:2601.18964) to appear in the Canadian Journal of Mathematics in collaboration with Karen Meagher.

CALVIN MORIER, York University
Fractional Revival Between Cospectral Vertices On Oriented Graphs

Continuous-time quantum walks have been studied for their potential applications to general-purpose quantum computing. Perfect state transfer (PST) occurs when the quantum walk transfers the state of one Qbit u to another Qbit v at some time. Periodicity occurs when the state of u is returned fully to u at some time.

Fractional revival (FR) interpolates between these two phenomena, occurring when the initial state of Qbit u is split among Qbits u and v according to the FR parameters $\alpha, \beta \in \mathbb{C}$, satisfying $|\alpha|^2 + |\beta|^2 = 1$:

$$U(\tau)\mathbf{e}_u = \alpha\mathbf{e}_u + \beta\mathbf{e}_v.$$

We study FR in the case of oriented graphs, using a skew-symmetric Hamiltonian matrix $H = iS$. If we further demand that the vertices u and v are cospectral, then particularly nice behaviour emerges: the FR will reoccur periodically, with parameters α and β changing each time according to a simple formula. Depending on the spectrum of the graph, this allows either periodicity at some multiple of the first FR time, or FR with parameters ϵ-close to arbitrary desired parameters at some multiple of the first FR time. Examples of both behaviours will be shown.

JOY MORRIS, University of Lethbridge
A Hilton-Milner theorem for unions of cliques

The Hilton-Milner theorem tells us about the maximum size for a family of intersecting sets, under the condition that the intersection of the sets is nonempty. Recall that the Erdős-Ko-Rado Theorem says that the absolute maximum size for a family of intersecting sets arises when the sets do all share a common element, so the Hilton-Milner theorem provides the best possible bound for any family that isn't just a subfamily of an EKR family. There is a significant gap between these bounds, which can prove very useful.

I will present more about this background, and discuss some recent results that involved producing a Hilton-Milner type theorem for independent sets in a union of cliques. This talk covers joint work with Karen Gunderson, Karen Meagher, Raghu Pantangi, and Mahsa Shirazi. Unfortunately for us, our results were pre-empted by a similar paper by Huajun Zhang and Huihui Zhang, which is available in *Discrete Math*.

RAGHU PANTANGI,
A generalization of the Erdős–Ko–Rado Theorem on permutations

Results analogous to the Erdős–Ko–Rado (EKR) theorem hold true for a wide range of mathematical objects that possess a notion of intersection. Among these is the analogue on permutations, characterizing the size and structure of maximum-sized families of pairwise intersecting permutations. Permutations $\sigma, \tau \in \mathrm{Sym}(n)$ are said to *intersect* if $\sigma^{-1}\tau$ fixes a point in $[n] := \{1, \ldots, n\}$. In this talk, we relax the definition of intersection and yet find that an analogue of EKR holds. Permutations $\sigma, \tau \in \mathrm{Sym}(n)$ are said to $\{1,2\}$-faux-intersect if either $\sigma^{-1}\tau$ fixes a point in $[n]$, or $\{\sigma^{-1}\tau(1), \sigma^{-1}\tau(2)\} \cap \{1,2\} \neq \emptyset$. We prove an EKR analogue for families of maximum-sized pairwise $\{1,2\}$-faux-intersecting permutations.

JOHNNA PARENTEAU, University of Regina

How Arbitrary is the Length of a Multiplicity List of a Graph?

Given a simple, undirected graph G on n vertices, let $S(G)$ be the collection of $n \times n$ real symmetric matrices $A = [a_{ij}]$ associated to G satisfying $a_{ij} \neq 0$ if and only if ij (when $i \neq j$) is an edge in G and the main diagonal is free to be chosen. For any square matrix A, let $q(A)$ equal the minimum number of distinct eigenvalues of A. The minimum number of distinct eigenvalues of G is defined as $q(G) = \min\{q(G) : A \in S(G)\}$. A natural question that arises from this association is: Does there exists a matrix $A \in S(G)$ with $q(A) = k$ for every integer k in the interval $[q(G), n]$? Since the inverse eigenvalue problem for graphs has been studied from many angles for years, a general solution seems difficult, so any progress made on subproblems, such as $q(G)$, contributes to an understanding of the whole problem. In this talk, I will discuss some recent progress on this questions for certain classes of graphs.

SAROBIDY RAZAFIMAHATRATRA, University of Lethbridge

On Ramanujan derangement graphs arising from linear groups of degree two

Let X be a connected non-bipartite k-regular graph on n vertices with eigenvalues $\lambda_1 > \lambda_2 \geq \ldots \geq \lambda_n$, and let $\lambda(X) := \max\{|\lambda_2|, |\lambda_n|\}$. The graph X is called a Ramanujan graph if $\lambda(X) \leq 2\sqrt{k-1}$.

Given a finite transitive group $G \leq \mathrm{Sym}(\Omega)$, the derangement graph Γ_G is the graph whose vertex set is G, and two elements g and h of G are adjacent if hg^{-1} is a derangement, that is, a fixed-point-free permutation. In this talk, we will show that Γ_G is a Ramanujan graph for every subgroup $\mathrm{SL}_2(\mathbb{F}_q) \trianglelefteq G \leq \mathrm{GL}_2(\mathbb{F}_q)$ acting transitively on the non-zero vectors of $\mathbb{F}_q^2$ such that $[\mathrm{GL}_2(q) : G]$ is odd.

This is based on joint work with Roghayeh Maleki.

ALIREZA TALEBPOUR, University of Regina

Average Mixing Matrix of Continuous Quantum Walks

The average mixing matrix is used to study the long-term average behavior of a continuous quantum walk on a graph. In this talk we will study the average mixing matrices of some important families of graphs including strongly regular graphs and hypercubes. We will also find out which of these matrices can be written as a linear combination of identity and all-ones matrices. This work is a product of the PIMS First Year Interest Groups (FYIG), joint with Joy Cooper, Hermie Monterde, Alireza Talebpour, and Ruth Wang.

HARMONY ZHAN, Worcester Polytechnic Institute