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Entrywise Powers and Critical Exponents of Graphs

Sparse covariance and correlation matrices motivate the study of entrywise transformations that preserve both positive semidefiniteness and a prescribed pattern of zeros. The Schur product theorem guarantees that positive integer entrywise powers preserve positive semidefiniteness, while a classical theorem of FitzGerald and Horn identifies a sharp threshold for real powers in each dimension. Prescribing a pattern of zeros leads to a finer problem in which the structure of a graph enters the picture.

The critical exponent of a graph is the threshold beyond which every entrywise power preserves positive semidefiniteness for matrices with nonnegative entries and zeros at the nonedges. Understanding this invariant involves determining its possible values and identifying the graph properties that govern it. This talk will introduce some classical results and discuss recent surprising progress on these questions. Small graphs and structured families will serve as concrete examples throughout, highlighting the interplay between graph structure and matrix positivity.