Dynamics and Symmetry
Dynamique et symétrie
(Org: **Archishman Saha** (University of Michigan) and/et **Cristina Stoica** (Wilfrid Laurier University))

STEPHEN ANCO, Brock University

LENNARD BAKKER, Brigham Young University
Dynamics and Generalized Symmetries

The symmetries of a flow play an important role in the analysis of its dynamics. Typical symmetries that a flow might possess are diffeomorphisms R that map the generating vector field X of the flow to $R_*X = \pm X$. A generalization of these symmetries are diffeomorphisms R that map X to $R_*X = \mu X$ for $\mu \in \mathbb{R}^\times \setminus \{1, -1\}$. We present theory and examples on the classification of equilibrium-free flows that possess these generalized symmetries.

CARLOS R. BARRERA-ANZALDO, Instituto Tecnológico Autónomo de México
Curving Lagrange points: more equilibria, more stability

We study the relative equilibria of the restricted circular three-body problem in two-dimensional spaces of constant curvature, which generalize the Lagrangian points $L_1, \ldots, L_5$. Fixing the Riemannian distance between the primaries, we provide rigorous proofs of phenomena observed numerically by Kilin (1999) and Martínez and Simó (2017). In positive curvature, new equilibria arise alongside the classical ones, whose existence is enforced by the compactness of the sphere. Finally, we show that positive curvature enlarges the range of mass ratios for which L_4 and L_5 are elliptic, and that one of the new equilibria is Lyapunov stable. This is joint work with Luis C. García-Naranjo and Miguel Ayala.

CONNOR JACKMAN, ITAM
Orbital symmetries of planar mechanical systems

An *orbital symmetry* of a planar mechanical system, $q'' = f(q)$, with $q \in \mathbb{R}^2$ is a plane transformation preserving the *unparametrized* orbits (a 3-parameter family of unparametrized plane curves for non-vanishing forces). In this talk we will present some examples and explain how the inverse problem: given the orbits, to reconstruct a mechanical system, is related to the orbital symmetry group and structure of integrals the system admits. This is work in progress joint with Gil Bor (CIMAT).

GIUSY MAZZONE, Queen's University
Vanishing limit of a ball in a rotating fluid-filled rigid body

We consider the physical system consisting of a hollow rigid body $\mathcal{B}$ whose cavity contains a homogeneous rigid ball B_R of radius R. Let the gap between $\mathcal{B}$ and B_R be entirely filled by a viscous incompressible fluid, and assume that the center of mass G of the whole system remains fixed during the motion. We are interested in the *vanishing limit* of the inner core B_R to a massless point as $R \to 0$, along solutions to the equations governing the *free rotations* of the whole system around G.

The equations of motion are given by the Navier–Stokes equations for the fluid and the balances of angular momentum for the two solids. We show that *weak solutions* to these equations converge (in an appropriate topology) to weak solutions of the equations governing the inertial motions of a fluid-filled rigid body (i.e., of the fluid-solid system without the inner core B_R). In addition, the long-time dynamics of the latter system can be recovered through the limit $R \to 0$ from the long-time dynamics of the system with the inner core B_R. These results show that the dynamics of a fluid-filled rigid body is robust under the singular perturbation obtained by inserting a small rigid core into the fluid-filled cavity.

PHILIP MORRISON, Univ of Texas at Austin

DAN OFFIN, Queen's University
Dynamics and Symmetry: Minimizing brake orbits

Reversible orbits play a special role in three body dynamics. Using Jacobi's version of the fixed energy variational problem, we prove that a geodesic in the Jacobi metric (JM) which touches the Hill boundary in two distinct points and which locally minimizes the energy functional (with certain restrictions) among curves which join at most two points on the boundary, but otherwise remain in the region where the JM is nondegenerate, correspond to hyperbolic periodic solutions of Newton's equations with energy E. This result holds in the nondegenerate case where the linearized equations have no periodic solutions other than the Hamiltonian vector field.

ARCHISHMAN SAHA, University of Michigan, Ann Arbor
Metriplectic Dynamics via Coupling to a Lie Group

Metriplectic systems provide a geometric formulation of non-equilibrium thermodynamics. They conserve total energy while a Casimir function, which plays the role of thermodynamic entropy, evolves monotonically. We describe a class of metriplectic systems obtained by coupling a Hamiltonian system to a Lie group via an equivariant momentum map. The dynamics couple a Hamiltonian flow with a double bracket (gradient) flow of the momentum map on a compact coadjoint orbit. Solutions of the metriplectic system can be constructed from the Hamiltonian flow by adding a geometric phase, and hence, the metriplectic contribution vanishes under projection to the symmetry-reduced manifold. We also show that the metriplectic entropy can be related to Shannon entropy by adding stochastic noise. As entropy production vanishes, the invariant measures of the stochastic processes converge to Souriau-Gibbs states on the coadjoint orbit at both finite and infinite (generalized) temperature. This is joint work in progress with Anthony Bloch.

NICOLA SANSONETTO, University of Verona
Geometric structure of certain nonholonomic systems with symmetry: integrability and Hamiltonisation

In Hamiltonian mechanics the conservation of the momentum map establishes a well understood relation between the presence of symmetries and the existence of conserved quantities. If constraints in the velocities are taken into account, the situation changes drastically: the variational and hence the Hamiltonian features typical of conservative holonomic mechanics are broken and in particular Noether's Theorem does not hold anymore. Besides that, in the last forty years there has been various approaches to extend Noether Theorem to the nonholonomic framework, and many interesting structures and results, as nonholonomic momentum map, nonholonomic reduction, Hamiltonisation and gauge momenta have been introduced and obtained. In this talk I will firstly recall the basic concepts of nonholonomic systems, symmetries, nonholonomic momentum maps, horizontal gauge momenta, and Hamiltonisation. Then I will give a result that allows to determine the number of horizontal gauge momenta that certain nonholonomic systems with symmetry may admit and also a method to compute them. I will then relate the existence of horizontal gauge momenta to the problem of Hamiltonisation and integrability of certain type systems.

This talk is based on an ongoing collaboration with Paula Balseiro from Fluminense Federal University - Brazil.

MANUELE SANTOPRETE, Wilfrid Laurier University

TANYA SCHMAH, University of Ottawa
Energy-conserving irreversible fluid models: a metriplectic approach

For modelling dissipative phenomena such as friction, diffusion and viscosity, the Hamiltonian framework has a key limitation: since Hamiltonian flows preserve phase-space volume, they admit no asymptotically stable equilibria, so typical solutions never relax to a steady state. The metriplectic framework modifies the Hamiltonian structure so that the flow still conserves energy but drives a chosen "generalised entropy" monotonically, allowing solutions to relax. We apply a version of this construction to ideal fluids. In 2D, dissipating the enstrophy (mean-square vorticity) yields the Navier-Stokes equations with an extra nonlocal term restoring energy balance; on the flat torus, at a fixed normalisation of the energy, these coincide with the constrained Navier-Stokes equations of Brzeźniak et al. In 3D, enstrophy cannot serve as the generalised entropy, since it is not conserved by ideal flow due to vortex stretching. Instead, for a fluid carrying an advected scalar such as temperature, we dissipate the "potential enstrophy", built from the vorticity and the temperature gradient. The result is a new energy-conserving alternative to the 3D Navier–Stokes equations, in which dissipation is nonlinear and anisotropic.

This is joint work with Erwin Luesink (Utrecht University), Archishman Saha (University of Michigan) and Cristina Stoica (Wilfrid Laurier University).