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Energy-conserving irreversible fluid models: a metriplectic approach

For modelling dissipative phenomena such as friction, diffusion and viscosity, the Hamiltonian framework has a key limitation: since Hamiltonian flows preserve phase-space volume, they admit no asymptotically stable equilibria, so typical solutions never relax to a steady state. The metriplectic framework modifies the Hamiltonian structure so that the flow still conserves energy but drives a chosen "generalised entropy" monotonically, allowing solutions to relax. We apply a version of this construction to ideal fluids. In 2D, dissipating the enstrophy (mean-square vorticity) yields the Navier-Stokes equations with an extra nonlocal term restoring energy balance; on the flat torus, at a fixed normalisation of the energy, these coincide with the constrained Navier-Stokes equations of Brzeźniak et al. In 3D, enstrophy cannot serve as the generalised entropy, since it is not conserved by ideal flow due to vortex stretching. Instead, for a fluid carrying an advected scalar such as temperature, we dissipate the "potential enstrophy", built from the vorticity and the temperature gradient. The result is a new energy-conserving alternative to the 3D Navier–Stokes equations, in which dissipation is nonlinear and anisotropic.

This is joint work with Erwin Luesink (Utrecht University), Archishman Saha (University of Michigan) and Cristina Stoica (Wilfrid Laurier University).