Combinatorial Design Theory
Théorie combinatoire du design
(Org: **Andrea Burgess** (University of New Brunswick Saint John) and/et **Shuxing Li** (University of Delaware))

JOSEPHINE BROOKS, University of Ottawa
A Recursive Approach to the Two-fold Oberwolfach Problem

The Oberwolfach problem OP$(m_1, \ldots, m_k)$ asks whether there is a decomposition of K_n into isomorphic 2-factors that are each isomorphic to a union of disjoint cycles of given lengths $m_1, \ldots, m_k$. A variant of the problem is the two-fold Oberwolfach problem OP$_2(m_1, \ldots, m_k)$, which asks for similar decompositions of the two-fold complete graph $2K_n$. The two-fold problem has been solved for some cases, including the uniform case with all equal cycle lengths, the case where each factor has two cycles, and the case where all cycle lengths are even, but in general the problem is still open. In this talk, we use a recursive approach to construct solutions to infinite families of larger cases of OP$_2$ from existing solutions to smaller cases of OP$_2$.

ANDREA BURGESS, University of New Brunswick
Colourings of group divisible designs and packing designs

A combinatorial design is a pair $(V, \mathcal{B})$ where V is a nonempty set of points, and $\mathcal{B}$ is a collection of subsets of $\mathcal{B}$, called blocks. A c-colouring of a design $(V, \mathcal{B})$ is a function $f : V \to C$ which assigns to each point one of c colours, such that each block contains at least two points of different colours. The design's chromatic number is the least value of c for which it admits a c-colouring. Colourings of balanced incomplete block designs have been extensively studied in the literature.

In this talk, we extend the study of colourings to group divisible designs and packing designs. We consider the existence of c-chromatic uniform k-GDDs with arbitrary group size. We further discuss colourings of GDDs with additional restrictions on the colours appearing in each group. We also present some results on block-equitable colourings of group divisible designs and packing designs; in this type of colouring, every colour must appear an equal number of times (or as closely as possible) in each block.

This is joint work with Peter Danziger, Diane Donovan, Tara Kemp, James Lefevre, David Pike and E. Šule Yazıcı.

AMANDA CHAFEE, Carleton University

PETER DANZIGER, Toronto Metropolitan University
A complete solution to the directed Oberwolfach problem of order 2 (mod 4) with cycles of even lengths

The Oberwolfach problem asks for a 2-factorization of the complete graph in which each 2-factor is isomorphic to a specific factor F. Recently, this problem has been extended to directed graphs. In this case, the directed Oberwolfach problem asks for a directed 2-factorization of the complete symmetric digraph in which each directed 2-factor is isomorphic to a specific directed factor F. We consider the directed Oberwolfach problem with directed 2-factors comprised of cycles of even lengths. We provide a complete solution to the case where F is a bipartite 2-factor and the order of the complete symmetric digraph is congruent to 2 modulo 4.

SEAN HAZEN, University of Delaware
Multiplicity Distribution of Quartic Polynomials over Finite Fields

In a 2020 paper by Li and Pott, the intersection distribution was introduced as an invariant of projectively equivalent $(q+1)$-subsets of the finite projective plane $PG(2, q)$, and can be equivalently characterized via polynomials over $\mathbb{F}_q$. The multiplicity

distribution was later introduced as a refinement. In this talk, we discuss previous work done to calculate the intersection and multiplicity distribution of polynomials over $\mathbb{F}_q$, as well as new results for the multiplicity distribution of arbitrary degree 4 polynomials over finite fields of characteristic 2 and 3.

ALICE MAN WA HUI, Clemson University

Subplanes of projective planes from collineation subgroups

A finite projective plane π of order n is a symmetric balanced incomplete block design (BIBD) with parameters 2-$(n^2+n+1, n+1, 1)$. The elements of the design are called points, and the blocks are called lines. A subplane of π consists of points and lines of π that form a projective plane under the incidence relation inherited from π. In this talk, we present sufficient conditions for the existence of subplanes constructed from subgroups of the collineation group of the ambient plane. We also count the subplanes obtained through these constructions.

This is joint work with Stephen Brittain and Robert Coulter.

MARY ROSE JERADE, University of Ottawa

The Honeymoon Oberwolfach Problem: A Recursive Approach

The Honeymoon Oberwolfach Problem (HOP) is a variant of the Oberwolfach Problem and was first introduced by Mateja Šajna. The problem asks whether it is possible to seat n couples attending a conference for $2(n-1)$ consecutive nights at ℓ round tables of sizes $2m_1, \ldots, 2m_\ell$ such that $m_i \geq 2$ for all $i = 1, \ldots, \ell$, $\sum_{i=1}^{\ell} m_i = n$, and each participant must be seated next to their spouse every night, but next to each other person exactly once. This problem is denoted by $HOP(2m_1, \ldots, 2m_\ell)$.

This seating arrangement problem smoothly translates into a graph theory problem. It is equivalent to finding a cycle decomposition of the graph $K_{2n} + (2n-3)I$ (that is, the complete graph K_{2n} with $2n-3$ additional copies of a fixed 1-factor I) into 2-factors such that each 2-factor consists of disjoint I-alternating cycles of lengths $2m_1, \ldots, 2m_\ell$. So far, solutions to HOP have been established for various cases, among which: for all $n \leq 20$; when all tables are of the same size; when the table sizes are divisible by 8; and when n is odd and the Oberwolfach Problem $OP(m_1, \ldots, m_\ell)$ has a solution.

In this talk, we present the latest solutions to this problem. The main finding is the following recursive result: given positive integers $m_1, \ldots, m_\ell, t$ such that $\sum_{i=1}^{\ell} m_i + a < t$, where a is the number of odd integers among $m_1, \ldots, m_\ell, t$, if there exists a solution for $HOP(2m_1, \ldots, 2m_\ell)$, then $HOP(2m_1, \ldots, 2m_\ell, 2t)$ also has a solution.

This is joint work with Mateja Šajna.

LAURA JOHNSON, University of Bristol

Using finite field cyclotomy to identify constructions of disjoint and external partial difference families

A disjoint (respectively, external) partial difference family, $S = \{D_1, \ldots, D_m\}$, is a collection of subsets of a group G such that each element $g \in \mathrm{G} \backslash \{0\}$ occurs λ times as a pairwise difference between elements of the same (respectively, distinct) subsets if $g \in \cup_{i=1}^{m} D_i$ and μ times if $g \notin \cup_{i=1}^{m} D_i$. In this talk, I will motivate the study of these objects and will also explore cyclotomic constructions of these structures.

Finite field cyclotomy concerns the study of cyclotomic classes and cyclotomic numbers in finite fields. Cyclotomic classes may be regarded as cosets of multiplicative subgroups of the non-zero elements of a finite field, while cyclotomic numbers provide information about the additive interactions between these multiplicative cosets.

As well as exploring cyclotomic constructions of disjoint and external partial difference families, I will present my contributions toward the explicit evaluation of cyclotomic numbers. I will also discuss future directions for developing new techniques to extend what is known about the evaluation of cyclotomic numbers in finite fields, with a view to using these techniques to identify new constructions of partial difference families.

WILLIAM KELLOUGH, Memorial University of Newfoundland

Searching for Balanced Incomplete Block Designs that Admit 0-ULSE Colourings

A uniform 0-locally semi-equitable (0-ULSE) ℓ-colouring of a balanced incomplete block design (BIBD) is a colouring of its points where within each block one colour is absent and the $\ell - 1$ other colours appear the same number of times. In this talk, we search for parameters (v, k, λ) where there exists a (v, k, λ)-BIBD that admits a 0-ULSE colouring. Joint work with Andrea Burgess, David Pike, and Thijs van Veluw.

TARA KEMP, Toronto Metropolitan University
Latin squares within latin squares

A latin square is an n by n array with each of n symbols occurring exactly once in each row and column, and a subsquare is a smaller latin square within a latin square. Given a partition $n = n_1 + n_2 + \cdots + n_k$, a realization is a latin square with disjoint subsquares of orders $n_1, n_2, \ldots, n_k$.

When there are at most four subsquares, the problem of existence of a realization has long been solved. This result has more recently been extended to five subsquares, and although it is tempting to consider six subsquares, at some point it is necessary to be bold and consider an arbitrary number of subsquares. As the first step into boldness, we prove a conjecture to be true: that if the largest three subsquares are exactly the same size, then such a realization always exists. Given that these realizations have an arbitrary number of subsquares, almost all of which could be of different sizes, the required constructions need to incorporate a large amount of flexibility. In this talk, we discuss some interesting parts of this construction, such as how to grow a subsquare.

DONALD KREHER, Michigan Technological University
Computing (λ, μ)-Factorizations of Abelian Groups

A (λ, μ)-factorization of a finite group G is a pair of subsets $S, T \subseteq G$ such that the product ST contains each nonidentity element of G exactly λ times and the identity element μ times. In group ring notation, this condition is expressed as

$$ST = \lambda(G - e) + \mu e,$$

where e denotes the identity element of G.

This presentation surveys theoretical results and focusus on computational methods for identifying such factorizations, including the complete enumeration of all factorizations of abelian groups of order up to 34.

This research is a collaborative effort with Shuxing Li and Doug Stinson.

ESTHER LAMKEN, Independent Researcher
A new type of orthogonality for t-designs

Let t, k, v be integers with $0 \leq t \leq k \leq v$. A Steiner system, $\mathsf{S}(t, k, v)$ is a pair $(X, \mathcal{A})$ where X is a v-element set and $\mathcal{A}$ is a set of k-subsets of X, called blocks, with the property that every t-element subset of X is contained in precisely one block of $\mathcal{A}$. If the blocks $\mathcal{A}$ of a Steiner system $\mathsf{S}(t, k, v)$ defined on X can be partitioned into $\mathsf{S}(i, k, v)$, then the Steiner system is called i-resolvable. In this talk, I will introduce β-orthogonality for i-resolvable Steiner systems and the associated arrays called (t, i, β)-Kirkman squares. Orthogonal packings in projective space can be used to construct the first infinite class of $(3, 2, \beta)$-Kirkman squares. These designs can be used to construct optimal doubly constant weight codes with perfect $(1, 2)$ covers, thus answering a long open question on the existence of such codes. This work is closely connected to several interesting open problems in design theory and finite geometry.

SHUXING LI, University of Delaware
Generalized Covering Radii of Melas Codes

Generalized covering radii are a recently introduced hierarchy of parameters for linear codes, motivated by their applications to database linear querying. They extend the classical covering radius and can be formulated as a natural subspace design

problem. Let $\mathcal{C}$ be a linear code over $\mathbb{F}_q$ with an $(n-k) \times n$ parity-check matrix H. The t-th generalized covering radius of $\mathcal{C}$ is the smallest positive integer r such that every t-subset of $\mathbb{F}_q^{n-k}$ lies in the span of some r columns of H. Exact values of these parameters are known for very few families of codes. Combine a counting argument with a careful character sum analysis, we obtain general lower and upper bounds on the generalized covering radii of Melas codes. As a consequence, we determine the second generalized covering radius of every Melas code.

This talk is based on joint work with Maosheng Xiong (Hong Kong University of Science and Technology).

ANDREW NAGARAJAH, Carleton University
Completing Large t-$(v, t+1, 1)$ Packings

At CMS this past summer, I gave a brief tour of various kinds of 'completion' phenomena for various kinds of (partial) combinatorial design, including a question whether t-$(v, t+1, 1)$ packings with sufficiently many blocks could be completed to a design. Here we give a sharp answer to this question by studying the leaves of such packings, and give an application to bounding packing numbers. We also present an interesting structural consequence of this result.

BUSHRA RAJA, Toronto Metropoiltan University
Weak 3-Coloring of Resolvable 4-Cycle System

A weak k-coloring of a resolvable m-cycle system $\mathcal{S}$ of order v (that is, a C_m-factorization of K_v) is an assignment of k colors to its vertices such that no m-cycle in any 2-factor is monochromatic; that is, every m-cycle receives at least two distinct colors. The weak chromatic number of $\mathcal{S}$ is the minimum number of colors k for which $\mathcal{S}$ admits a weak k-coloring.

While there has been substantial research on colorings of m-cycle systems (that is, C_m-decompositions of K_v), relatively little attention has been given to weak colorings of resolvable m-cycle systems. In this talk, we consider weak 3-colorings of resolvable 4-cycle systems.

This is joint work with Dr. Peter Danziger and Dr. Andrea Burgess.

MATEJA ŠAJNA, University of Ottawa
A recursive approach to the Oberwolfach problem and its variants

The celebrated Oberwolfach problem (OP), first posed by Ringel in 1967, asks whether n participants at a conference can be seated at k round tables of sizes $m_1, \ldots, m_k$ (where $m_1 + \ldots + m_k = n$) for several meals so that everybody sits next to everybody else exactly once. OP can be modeled as a decomposition of the complete graph K_n into 2-factors, each consisting of k disjoint cycles of lengths $m_1, \ldots, m_k$.

For cycles of uniform length, OP was solved decades ago, and it has recently been shown that it has a solution for all sufficiently large n. In addition, many infinite families of cases with variable cycle lengths (e.g. the two-table case and the bipartite case) are known to have a solution, however, in general, OP is still open and highly relevant. Moreover, many variants, each modeling a different seating arrangement, have been investigated recently.

In this talk, I will describe a recursive approach that allows us to use a solution to a smaller case of OP or variant to construct a solution to a larger case (typically adding one sufficiently long cycle). This approach has already been used successfully to obtain new solutions to several infinite families of cases of the directed OP (for complete symmetric digraphs), and I will outline my plans for some of the other variants. This talk will also serve as an introduction to the talks by Marie Rose Jerade and Josephine Brooks.

KIANOOSH SHOKRI, University of Ottawa
New construction of covering perfect hash families and covering arrays

In this talk, we will show a very simple and powerful construction of covering perfect hash families that improves bounds on many covering array numbers. This is a very recent result from joint work with Lucia Moura and Brett Stevens.

BRETT STEVENS, Carleton University
Embedding a graph in a Steiner Triple System?

Since every pair of vertices appear in a block of a Steiner Triple System, what would it even mean to embed a graph in a STS; after all every graph is a subgraph of a complete graph. I will discuss more meaningful ways to interpret the idea of embedding a graph and discuss when embeddings exists, bounds on their sizes and small embeddings. One interpretation connects to the combinatorial game Node Kayles and its hypergraph generalization NoFil.

DOUGLAS STINSON, University of Waterloo
Optimal Equidistant Codes with Alphabet Size $q \geq 3$

There has been recent renewed interest in optimal equidistant codes; however, research on this topic goes back almost 60 years. At the most recent Summer CMS meeting, I spoke on optimal equidistant codes over a binary alphabet. Here I will turn to codes over an alphabet of size $q \geq 3$. First, I will show an equivalence between equidistant codes and (r, λ)-designs. This equivalence is fairly obvious, but it doesn't seem to have been exploited in the research literature. Using this equivalence, I will discuss some new and old results on equidistant codes. I will show how some of results can be proven easily using results on (r, λ)-designs, by Vanstone and others, that date back to the 1970's. This talk is based on joint work with Tim Alderson.