MARY ROSE JERADE, University of Ottawa

The Honeymoon Oberwolfach Problem: A Recursive Approach

The Honeymoon Oberwolfach Problem (HOP) is a variant of the Oberwolfach Problem and was first introduced by Mateja Šajna. The problem asks whether it is possible to seat n couples attending a conference for $2(n-1)$ consecutive nights at ℓ round tables of sizes $2m_1, \ldots, 2m_\ell$ such that $m_i \geq 2$ for all $i = 1, \ldots, \ell$, $\sum_{i=1}^{\ell} m_i = n$, and each participant must be seated next to their spouse every night, but next to each other person exactly once. This problem is denoted by $HOP(2m_1, \ldots, 2m_\ell)$.

This seating arrangement problem smoothly translates into a graph theory problem. It is equivalent to finding a cycle decomposition of the graph $K_{2n} + (2n-3)I$ (that is, the complete graph K_{2n} with $2n-3$ additional copies of a fixed 1-factor I) into 2-factors such that each 2-factor consists of disjoint I-alternating cycles of lengths $2m_1, \ldots, 2m_\ell$. So far, solutions to HOP have been established for various cases, among which: for all $n \leq 20$; when all tables are of the same size; when the table sizes are divisible by 8; and when n is odd and the Oberwolfach Problem $OP(m_1, \ldots, m_\ell)$ has a solution.

In this talk, we present the latest solutions to this problem. The main finding is the following recursive result: given positive integers $m_1, \ldots, m_\ell, t$ such that $\sum_{i=1}^{\ell} m_i + a < t$, where a is the number of odd integers among $m_1, \ldots, m_\ell, t$, if there exists a solution for $HOP(2m_1, \ldots, 2m_\ell)$, then $HOP(2m_1, \ldots, 2m_\ell, 2t)$ also has a solution.

This is joint work with Mateja Šajna.