Commutative Algebra
Algèbre commutative
(Org: **Giulia Gaggero**, **Lisa Seccia** and/et **Adam Van Tuyl** (McMaster University))

KIERAN BHASKARA, McMaster University
The GVD property of toric edge ideals and binomial edge ideals

Geometrically vertex decomposable (GVD) ideals form a class of ideals that generalize the Stanley-Reisner ideals of vertex decomposable simplicial complexes. Recent work has investigated the GVD property in the context of toric ideals of graphs. In this setting, several families of graphs with GVD toric ideals have been identified, as well as connections between the GVD property and other algebraic invariants. However, a complete characterization of GVD toric ideals remains unknown. In this talk, we present a new family of graphs whose toric ideals are GVD. We also initiate a study of the analogous problem for binomial edge ideals, and describe two families of graphs whose binomial edge ideals are GVD. We show that all Cohen-Macaulay binomial edge ideals for graphs with fewer than 12 vertices are GVD, and make a conjecture about the occurrence of such ideals for larger graphs. This talk is based on joint work with Adam Van Tuyl and Sasha Zotine, and work in progress with Akihiro Higashitani, Lisa Seccia, and Adam Van Tuyl.

MICHAEL BURR, Clemson University
Equivalences between Subalgebra and Khovanskii bases

Subalgebra bases and Khovanskii bases are analogues of Gröbner bases in the setting of algebras. We will show, in the most common cases, how to transform Khovanskii bases into subalgebra bases (and vice versa). We will see that this transformation preserves the structure and key data of Khovanskii and subalgebra bases. Moreover, this equivalence allows us to translate computations with Khovanskii bases to computations in existing computer algebra systems with subalgebra bases.

ENRICO CARLINI, Politecnico di Torino
Minimal apolar subsest to homogeneous polynomials

We will briefly recall some basic facts about sum of powers decompositions, Waring rank, and apolarity. Then we will explore some recent results with Bhat-Dubey-Masuti on minimal apolar subsests to some reducible quartics. In particular, we will see how spiders and butterflies can explain the geometry of sum of powers decompostions of $x^2(z_1^2 + \ldots + z_m^2)$ and of $(x^2 + y^2)(z_1^2 + \ldots + z_m^2)$.

SUSAN COOPER, University of Manitoba
Splitting Polyomino Ideals

Computing the graded Betti numbers of homogeneous ideals in polynomial rings is central to numerous open problems. We often restrict to families of ideals generated by combinatorial objects in order to gain additional insight. In this talk, we investigate polyomino tilings to generate monomial ideals. We apply the theory of splittings to obtain a recursive formula for the graded Betti numbers of these ideals. This is joint work with Serena Werbowesky.

SELVI KARA, Bryn Mawr College
Edge ideals of gapfree graphs with matching number 3

Powers of edge ideals often reflect the combinatorics of the underlying graph. Nevo and Peeva conjectured that sufficiently high powers of the edge ideal of a gapfree graph have linear resolutions. Previous work on this conjecture has often imposed additional algebraic hypotheses. In this talk, I will discuss an approach based instead on combinatorial restrictions, using the

matching number to organize the problem. We focus on gapfree graphs with matching number three and show that every power of their edge ideals starting with the second has linear quotients and hence a linear resolution. Building on our previous work, we combine graph-theoretic reductions with operations that preserve linear quotients to reduce the proof to 26 graphs on at most seven vertices. I will explain these reductions and discuss how this approach may guide further progress on the conjecture. This is joint work with Nursel Erey, Tài Huy Hà, Takayuki Hibi, and Susan Morey.

GRAHAM KEIPER, The University of Catania
OBSTRUCTIONS TO EQUALITY OF THE SECOND SYMBOLIC AND REGULAR POWERS OF TORIC IDEALS OF GRAPHS

I will present recent joint work with Elena Guardo and Grzegorz Malara. Let G be a bipartite finite simple graph and let I_G be its associated toric ideal. We will discuss recent progress made towards characterizing obstructions to the equality between the second symbolic power, $I_G^{(2)}$, and the second regular power, I_G^2.

BRETT NASSERDEN, McMaster University
Frobenius splittings of certain K-orbit closures

Matrix Schubert varieties are orbit closures for the action of two Borel subgroups on matrix space. In positive characteristic, the standard Frobenius splitting constructed via Bott–Samelson varieties induces a splitting of matrix space, realized as a Bruhat cell, that preserves every matrix Schubert ideal. We use Barbasch–Evens–Magyar varieties to study the analogous splitting problem for orbits of $\mathrm{GL}_a \times \mathrm{GL}_b$ and Sp_{2n} on flag varieties. In the symplectic case, we apply these methods to study skew-symmetric matrix Schubert varieties and their squarefree Gröbner degenerations.

NAVEENA RAGUNATHAN, McMaster University
The Lefschetz Properties of Artinian Rings Associated to the van der Waerden Simplicial Complex

The Lefschetz Properties are a well-studied characteristic of Artinian rings, with its history tracing back to Stanley's study of the Lefschetz Properties on complete intersections. In this talk, I will discuss the occurrence of the Lefschetz Properties on Artinian rings created from the Stanley-Reisner ring of the van der Waerden simplicial complex, further quotiented out by the squares of all variables. These simplicial complexes, defined on positive integers n>k, are generated by all arithmetic progressions of size k on n variables. We prove that for the smallest and largest possible values of k, namely k=1,2, and n-1, we always have the occurrence of the Weak Lefschetz Property. We also examine the first occurrence of failure at k=3, and prove that this failure propagates for all higher n values. I will also give a complete characterization of when our simplicial complex is a pseudo-manifold, and the implications this has on the occurrence of the Weak Lefschetz Property in certain degrees. Finally, I will present some recent ideas from joint work with Dr. Brett Nasserden, in which we generalize the failure of the k=3 case to all odd values of k, and examine the case of characteristic p.

REBECCA RG, George Mason University
Classifying neural codes using algebraic tools

The neural ideal was introduced by Curto, Itskov, et al in 2013 to study the spatial firing patterns of a set of neurons, turning problems in neuroscience and coding theory into algebraic questions. In this talk I will show how homological invariants such as Betti numbers can be applied to study and classify neural ideals. This work is joint with Hugh Geller, Alexandra Seceleanu, and Nora Youngs.

OLIVIA STRAHAN, University of New Mexico
Local cohomology in t-Stanley-Reisner rings

There is a profound relationship between the algebraic invariants of Stanley-Reisner rings and the topological data of the associated simplicial complex. Hochster's formula, which describes the local cohomology modules in terms of the simplicial

cohomology of links, is one of many famous theorems built on this relationship. This talk will introduce a mixed-characteristic analogue of Stanley-Reisner theory and the adaptation of Hochster's formula to this new setting. The adaptation uses "cellular sheaf cohomology," wherein different coefficients are assigned to each face. These and related results are from a joint paper with Mel Hochster.

KESAVAN MOHANA SUNDARAM, University of Nebraska-Lincoln
Annihilators of local cohomology in unramified mixed characteristic

Annihilators of local cohomology modules play an essential role in the proofs of many fundamental results in the homological commutative algebra. In 2019, Hochster asked whether the top local cohomology module of a domain, with support in a given ideal, must be faithful. In 2020, Hochster and Jeffries gave an affirmative answer for domains of positive characteristic when the cohomological dimension of the ideal equals its minimum number of generators. Later, in mixed characteristic, Datta, Switala, and Zhang (2023) constructed a counterexample to Hochster's question, showing that faithfulness can fail even for regular local rings of mixed characteristic. In this talk, we compute the annihilators of local cohomology modules with support in ideals associated with standard triangulations of certain topological spaces. We will also see some partial progress on answering Hochster's question in characteristic zero.

VASU TEWARI, University of Toronto
Power ideals and symmetric group representations

I will discuss zonotopal algebras: a family of quotients of the polynomial rings by certain power ideals, as defined by Ardila and Postnikov (and several others). I will then relate them to representation theory of symmetric groups by appealing the method of "orbit harmonics" and discuss some applications. Joint work with Brendon Rhoades and Markus Reineke.