AARMS-CMS Student Poster Session
Présentations par affiches des étudiants - AARMS-SMC

KEVIN ANDERSON, University of Ottawa
Towards a Second Fundamental Theorem for $U_q(\mathfrak{gl}_n)$

Classical invariant theory studies the elements of a polynomial algebra which are invariant under a given group action. An answer to this problem consists of two parts: a "First Fundamental Theorem" (FFT) of invariant theory describes the generators of the invariant subalgebra, and a "Second Fundamental Theorem" (SFT) describes the relations among these generators. The same problem can be posed in other contexts in algebra and representation theory. For instance, one can replace the group with a Hopf algebra and consider actions (and coactions) on a quantized polynomial algebra. Recent work by Letzter–Sahi–Salmasian (2024) has established an "operator commutant" FFT for the quantized enveloping algebra $U_q(\mathfrak{gl}_n)$ in such a setting. The relevant polynomial algebra considered is a quantized Weyl algebra, which is constructed as a deformed twisted tensor product and comes with a natural $U_q(\mathfrak{gl}_m) \otimes U_q(\mathfrak{gl}_n)$-module algebra structure. In this case, the invariant subalgebra is generated by certain homogeneous degree 2 polynomials called quantized polarization operators. Our present research aims to better understand the structure of this invariant subalgebra, first and foremost by determining the commutation relations among these polarization operators.

PEDRO BAPTISTA, Universidade Federal de Minas Gerais
Compact graphs and quantum automorphisms

Compact graphs are graphs for which the fractional automorphism polytope has no genuinely fractional vertices. This paper proposes a quantum analogue of this idea by evaluating the fundamental magic unitary of the quantum automorphism group on states, which we show to produce a closed convex set of doubly stochastic matrices sitting between the classical automorphism polytope and the full fractional automorphism polytope. Our main result is that the natural quantum analogue of compactness is classical, that is, a quantum compact graph is classically compact. We also relate this set to the quantum orbital algebra and obtain a hierarchy of classical and quantum compactness pseudo notions. The framework recovers familiar consequences of compactness through commutants and suggests quantum analogues of generous transitivity and distance-transitivity. We also isolate examples and open problems indicating where quantum symmetries may strictly refine the classical compactness theory.

LIVIA ALAMINO CAETANO, Universidade Federal de São Carlos (UFSCar)
Fuzzy logic applied to the prediction of Attention Deficit Hyperactivity Disorder based on alterations in brain structures

The brain is the most complex organ of the human body, capable of making thousands of connections and controlling fundamental functions of the body. The Diagnostic and Statistical Manual of Mental Disorders (DSM) brings together the definition and classification of different mental disorders, including Attention Deficit Hyperactivity Disorder (ADHD). Although specific causes of the disorder have not been identified, there is evidence of anatomical differences in the brains of people with ADHD compared to those without the disorder. The goal of this scientific research was to analyze how different structural changes in the brain can contribute to the manifestation of ADHD For this, a Fuzzy Rule-Based System (FRBS) was built with input variables White Matter, Cerebral Cortex, Caudate Nucleus, and Total Brain Volume, all of them characterized with the linguistic terms appropriate, partially appropriate, and slightly appropriate. The output variable considered was the Structural Index, classified into significant variation, moderate variation, and low variation. The inference method used was Mamdani's, and defuzzification was carried out using the Center of Gravity method. Simulations of clinical scenarios of hypothetical patients were implemented in MatLab software and the results achieved show that the fuzzy model is capable of providing relevant information about ADHD diagnosis. The next steps in the research involve also considering functional variables, in a second model and combining both models. It should be emphasized that mathematics and fuzzy logic are suitable tools to assist in modeling real phenomena, including in medicine, and can contribute to decision-making in complex situations.

JADE DAVIES, University of Victoria
Intersection Numbers of Sudoku Latin Squares

Let $n = hk$, where h and k are integers with $h, k \geq 2$. We study the set of possible intersection numbers of $n \times n$ latin squares having the additional 'Sudoku' constraint based on an $h \times k$ grid of $k \times h$ boxes.

DANIEL FASSLER, Concordia University
Randomized Sketching for Large-Scale Spectral Recovery: The MAM^* Method and New Directions

The past decade has seen an explosion in the availability of experimental datasets and the rise in personal and professional computing power. For numerical simulations, this enables finer space resolution over longer time horizons, yielding data sets that are costly to store and process. This motivates techniques such as randomized sketching, which offers structure-preserving dimension reduction of data sets and can enable singe-pass or streaming data processing. In this poster, we first review the $MAM*$ framework, a recent development in randomized sketching for low-rank approximation and spectral recovery. This approach combines randomized linear measurements with compressed sensing techniques to recover sparse approximations of the leading eigenvectors of extremely large, approximately low-rank matrices. Building on these developments, we outline ongoing research directions aimed at extending randomized sketching techniques to the recovery of singular vectors and low-rank factors of large matrices and tensors.

JOEY FINGOLD, University of Guelph
A statistical framework for modelling underreported infectious diseases

Count time series models are popular for modelling surveillance data of infectious diseases, such as COVID-19 and influenza, due to their flexibility and interpretability. However, observed data on daily infections is subject to underreporting, as not every infected individual seeks medical care. Such a model is challenging to fit since it involves high-dimensional and serially correlated integer-valued unknowns. One solution is to consider an analogous continuous-valued surrogate model whose values are post-hoc mapped to integers. This procedure produces biased estimates as inference is performed using samples from such a surrogate model and not the thinned count autoregression itself. We propose a sequential importance sampling procedure to correct this misspecified model. We demonstrate its validity in a simulation study and its applicability for epidemic curve reconstruction using rotavirus data from Germany.

EMANUEL JULIANO, Federal University of Minas Gerais
Tightness of a MaxCut Lower Bound via Vector Chromatic Number

Recently, Balla, Janzer, and Sudakov showed a lower bound on the MaxCut in terms of the vector chromatic number, recovering known results on the MaxCut of H-free graphs. We show that their bound is tight, providing a construction that achieves a value arbitrarily close to the optimal constant. This answers a question raised by Elphick. Our construction is a modification of the geometric graph used by Feige and Schechtman to establish the integrality gap for the Goemans–Williamson semidefinite relaxation of the MaxCut.

LAURENT LIZOTTE, Université de Montréal
Atomic Decomposition of Non-Homogeneous Hardy Spaces

Real Hardy spaces emerge in analysis as suitable substitutes of L^p spaces when p is less than 1. Functions in these spaces can be nicely decomposed into a sum of basic building blocks. This work generalizes these so called *atomic decompositions* to *non-homogeneous* Hardy spaces defined on certain metric measure spaces. We will start by defining the function spaces in play, noting the differences with the global Euclidean case; then we will present the modified atomic decomposition and some geometric lemmas used to prove the result. This project was supervised by Denis Vinokurov (Université de Montréal) and Galia Dafni (Concordia University).

CANSU ÖZDEMIR, University of Toronto
Generalized Large Algebras for Lie Algebra Contractions 101

İnönü-Wigner contractions were introduced in 1953 to explain how the symmetries of two systems connected by a limit are related; for example Poincaré group contracts to Galilei group as the speed of light goes to infinity. During an informal talk, Cahit Arf asked Erdal İnönü whether there is a large group that includes both the original and the contracted group as subgroups. İnönü answered this question in 1956 by constructing a large algebra, a Lie algebra containing both the original and the contracted algebra as subalgebras, and a corresponding large Lie group. However, this only covers first order contractions, where $\mathfrak{m}$ is rescaled by ε, for a decomposition $\mathfrak{g} = \mathfrak{s} \oplus \mathfrak{m}$ into a subalgebra $\mathfrak{s}$ and a complement $\mathfrak{m}$ satisfying İnönü's condition $[\mathfrak{s}, \mathfrak{m}] \subset \mathfrak{m}$. It does not cover higher order (Weimar-Woods) contractions, where generators are rescaled by powers ε^k with non-negative integer exponents k, the largest being the order n. In 2011, Khasanov and Kuperstein introduced a new approach to İnönü-Wigner contractions through quotients of the Lie algebra of smooth sections on the İnönü-Wigner Lie algebra bundle, and defined a sequence of higher dimensional Lie algebras $\mathfrak{g}_n$. They study $\mathfrak{g}_1$, but not $\mathfrak{g}_n$ for $n \geq 2$ in general. This poster aims to show that for the trivial Lie algebra bundle of $\mathfrak{g}$, every Weimar-Woods contraction of order n is a canonical subquotient of $\mathfrak{g}_n = \mathfrak{g} \otimes \mathbb{R}[\sigma]/(\sigma^{n+1})$, so that $\mathfrak{g}_n$ plays the role of a large algebra for contractions up to order n.

SARA PALIANI, Concordia University
Cantor sets, BV functions and BMO-type norms

We study functions of bounded variation (BV) through the classical Cantor-Lebesgue function (also known as the Devil's Staircase), which is both a fractal curve and an example of a one-dimensional BV function. Moreover, every BV function can be decomposed into three parts, one of which is Cantor-like. The goal is to explore concretely the relation between BV functions and BMO-type norms in the one-dimensional case by computing and comparing such norms in the case of the Cantor function. This is inspired by recent work of various authors attempting to compute the total variation from the BMO-type norms by using various notions of convergence such as Gamma-convergence.

JOHNNA PARENTEAU, University of Regina
Small Spectra are Spectrally Arbitrary for Threshold Graphs

Given a simple, undirected graph G on n vertices, let $S(G)$ be the collection of $n \times n$ real symmetric matrices $A = [a_{ij}]$ associated to G satisfying $a_{ij} \neq 0$ if and only if ij (when $i \neq j$) is an edge in G and the main diagonal is free to be chosen. For any square matrix A, let $q(A)$ equal the minimum number of distinct eigenvalues of A. The minimum number of distinct eigenvalues of G is defined as $q(G) = \min\{q(G) : A \in S(G)\}$. For the class of graphs known as *threshold graphs*, it is known that $q(G) \leq 4$. We present an alternative proof of this result and show that, for any four distinct numbers, there exists a matrix in $S(G)$ whose eigenvalues realize that sequence.

WILLIAM DANIEL STEPHENSON, University of Ottawa
Solutions to the parabolic Anderson model on an open domain

The parabolic Anderson model is a stochastic partial differential equation that describes diffusion in a random environment. We consider the model on an open domain $\mathcal{D}$ with Dirichlet boundary conditions, and noise given by an isonormal Gaussian process that is fractional in time. We show that the existence of a mild solution on the full space implies existence to the corresponding equation on $\mathcal{D}$. Our approach relies on approximating the solution by random fields that admit a Feynman–Kac representation.

SOFIA MEDINA VARELA, University of Regina
Representations of Noncommutative Cubes and Prisms

Representations of the operator system determined by the canonical generators of the free product of two cyclic groups of order 2 and k, or d cyclic groups of order 2, are studied for the purpose of shedding light on the noncommutative geometry

of noncommutative d-cubes and k-prisms. By way of the duality of the categories NCConv and OpSys of noncommutative convex sets and operator systems, respectively, an analysis of noncommutative extreme points, exactness, the lifting property, automatic complete positivity, controlled completely positive extensions, tensor products, and operator system duality is undertaken. Of note is the pairing of two classical dilation theorems of Halmos and Mirman to give a complete description of the noncommutative triangular prism in terms of joint unitary dilations.

CÉCILE VERRIER, Univ. Lille, CNRS, INRIA, UMR 8524 – Laboratoire Paul Painlevé, F-59000 Lille, France
Mathematical Modeling of Senescent Cells in Cancer Treatment

Cancer is a complex disease, the progression of which depends on interactions between tumor cells and their environment. Among the mechanisms involved, cellular senescence, which leads to a permanent halt in cell proliferation, has long been recognized for its protective role against cancer. However, studies have revealed that senescent cells can also promote tumor progression and limit the efficacy of treatments, notably by altering the immune response. What if these two effects were to coexist, and certain unexpected treatment combinations could allow us to harness senescent cells to our advantage, rather than suffering from their harmful effects?

These cells secrete molecules that can support the tumor, but they can also facilitate the action of therapeutic treatments. These molecules are known as the SASP (senescence-associated secretory phenotype), and it is this duality that we aim to understand using a mathematical model.

To gain further insight, we are developing an ordinary differential equation (ODE) model that describes the interactions between senescent cells, tumor cells, the immune system, and blood vessels. Calibrated using experimental data from mice, this model will enable us to explore the efficacy of various therapeutic combinations, depending on the specific biological context.

JINXIN WANG, University of Ottawa
A Functional Central Limit Theorem for the KPZ Equation with Narrow Wedge Initial Data

The Kardar–Parisi–Zhang (KPZ) equation is a fundamental model for surface growth. I will begin by introducing the equation and its connection with the stochastic heat equation via the Cole–Hopf transform. Building on a recent finite-dimensional convergence result, I will then present our proof of tightness, which yields a functional central limit theorem for spatial averages of the KPZ solution with narrow wedge initial data. The proof combines the Clark–Ocone formula, a common-ray change of spatial variables, and a Chapman–Kolmogorov relation to obtain uniform moment bounds for time increments on compact time intervals bounded away from zero. This is joint work with Raluca M. Balan and Le Chen.

SHUOYANG WANG, McMaster University
Stationary States on Periodic Metric Graphs with Nonzero Boundary Conditions

Stationary states of the nonlinear Schödinger (NLS) equation are considered on periodic metric graphs. Compared to the existing literature, we study stationary states with nonzero boundary conditions which model black solitons in optics. For a general graph with a single edge connecting compact cores symmetrically and periodically, we prove the existence of an energy minimizer pinned to the point of the edge symmetry. For the neckless graph example, we prove the coexistence of two energy minimizers pinned to two points of symmetry: one is in the middle of the link connected to two loops and the other one is in the middle of the loop connected to two links. In the semi-classical limit, which is equivalent to the limit of long edges, we prove that no other stationary states exist, whereas the link- pinned state is a local minimizer of energy and the loop-pinned state is a saddle point of energy without the pinning condition. We also prove that both are unstable in the time evolution of the NLS equation in such a way that the energy minimizer is strongly unstable with a real eigenvalue and the saddle point of energy is weakly unstable with a quadruplet of complex eigenvalues. The analytical results are illustrated away from the semi-classical limit by using numerical approximations.