**Association schemes and applications**
**Les schémas d'association et applications**
(Org: **Roghayeh Maleki** (University of Regina) and/et **Sarobidy Razafimahatratra** (University of Lethbridge))

**ROBERT BAILEY**, Memorial University of Newfoundland

**PIERRE-ANTOINE BERNARD**, University of Toronto

**DOUGLAS FARENICK**, University of Regina
*Matrices associated with unitary representations of the modular group*

Finite-dimensional irreducible unitary representations of the modular group–when viewed as the free product of cyclic groups of orders 2 and 3, respectively–give rise to certain pairs of complex matrices by way of noncommutative convex combinations of matrix pairs arising from such representations of the two canonical order-2 and order-3 generators of the modular group. It turns out that all matrix pairs occurring as such noncommutative convex combinations can be determined by a property of their joint numerical range. At the heart of this result, which may be understood to be a pairing of two classical dilation theorems (Halmos 1950; Mirman 1968), is a 4-dimensional complex vector space that shares some, but not all, of the defining features of an association scheme. Based on joint work with Roghayeh Maleki, Sofia Medina Varela, and Sushil Singla.

**CHRISTOPHER FRENCH**, Grinnell College
*Metathin association schemes and hypergroups*

When one forgets the points of a thin association scheme, one obtains a group. For an arbitrary association scheme, forgetting the points leads to a $hypergroup$, an algebraic structure analogous to a group, but where the product of two elements (relations) is a set of elements, rather than a single element. One interesting class of association schemes is the $metathin$ association schemes – these are those which have a thin residue which is itself thin, and are therefore in a sense built up from a pair of groups. Metathin hypergroups are defined in an analogous way. Then, the underlying hypergroup of a metathin association scheme is a metathin hypergroup, and in fact, one can recover the structure constants of the association scheme from which the hypergroup arose. It is then natural to wonder if every metathin hypergroup can be obtained from an association scheme. In this talk, we present examples of metathin hypergroups which $cannot$ be obtained as the underlying hypergroup of an association scheme.

**ALLEN HERMAN**, University of Regina
*On the second largest eigenvalue of certain graphs in the perfect matching association scheme*

The perfect matching association scheme $\mathcal{M}_{2n}$ is the orbital scheme of $Sym(2n)$ acting on the cosets of a hyperoctahedral subgroup of type $B_n$. Its' relations and eigenspaces are naturally indexed by the even partitions of $2n$ ($2\lambda$ for $\lambda \vdash n$), and the first eigenmatrix of the scheme is integral. It thus shares many parallels with the conjugacy class scheme $cc(S_n)$, whose rows and columns are naturally indexed by the partitions of $n$ and whose first eigenmatrix is integral. While the eigenvalues of $cc(S_n)$ can be computed using values of the symmetric functions $s_\lambda(\mu)$, the eigenvalues of $\mathcal{M}_{2n}$ are less understood.

The talk will discuss recent progress on a conjecture of Meagher concerning the second largest eigenvalue of $A_{2\lambda} \in \mathcal{M}_{2n}$: if the partition $\lambda$ has at least two $1$'s,then the second largest eigenvalue of $A_{2\lambda}$ occurs on the $2(n-1,1)$-eigenspace. This conjecture is analogous to the Aldous property for graphs in $cc(S_n)$, proved by Li, Xia, and Zhou in 2023: when $\lambda$ corresponds

to the conjugacy class of a permutation with at least two fixed points, then the second largest eigenvalue of $A_\lambda \in cc(S_n)$ occurs on the $(n-1,1)$-eigenspace.

This is joint work with Himanshu Gupta, Alice Lacaze-Masmontiel, Roghayeh Maleki, and Karen Meagher.

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**SHUXING LI**, University of Delaware

*Strongly regular graphs with generalized Denniston parameters*

In 1969, Denniston [2] introduced a family of maximal arcs in Desarguesian planes of even order, a construction that gives rise to a classical family of partial difference sets—objects that correspond to strongly regular graphs and have deep connections to finite geometry. These partial difference sets, later named after him, are defined within the additive group of a finite field of characteristic 2.

This naturally raises the question: Do Denniston partial difference sets exist in fields of odd characteristic? For over five decades, no progress was made on this problem—until recent breakthroughs by multiple research groups established the construction of Denniston partial difference sets in elementary abelian groups [1, 3, 4].

Building on this momentum, we extend Denniston partial difference sets to a significantly broader class of elementary abelian groups [5], which lead to a family of strongly regular graphs with generalized Denniston parameters. This is joint work with James Davis, Sophie Huczynska, Laura Johnson, and John Polhill.

# References

[1] J. Bao, Q. Xiang, and M. Zhao. Partial difference sets with Denniston parameters in elementary abelian $p$-groups. *Finite Fields and Their Applications*, 2025.

[2] R.H.F. Denniston. Some maximal arcs in finite projective planes. *Journal of Combinatorial Theory*, 1969.

[3] J.A. Davis, S. Huczynska, L. Johnson, and J. Polhill. Denniston partial difference sets exist in the odd prime case. *Finite Fields and Their Applications*, 2024.

[4] S. De Winter. Projective two-weight sets of Denniston type. *Combinatorial Theory*, 2025.

[5] S. Li, J.A. Davis, S. Huczynska, L. Johnson, and J. Polhill, Strongly regular graphs with generalized Denniston and dual generalized Denniston parameters. *Combinatorial Theory*, 2026.

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**ROGHAYEH MALEKI**, University of Regina

*Terwilliger algebras and triple transitivity in quasi-thin association schemes*

An association scheme $\mathfrak{X}$ on a finite set $V$ is a partition of $V \times V$ satisfying certain regularity conditions. The Terwilliger algebra $T_v$ of $\mathfrak{X}$ with respect to a vertex $v \in V$ is the algebra generated by the adjacency matrices of $\mathfrak{X}$ and the diagonal projections onto the subconstituents relative to $v$. The algebra $T_v$ satisfies the inclusions

$$T_{0,v} \subseteq T_v \subseteq \widetilde{T}_v,$$

where $T_{0,v}$ is a certain subspace, and $\widetilde{T}_v$ is the centralizer algebra of the stabilizer of $v$ in the automorphism group $Aut(\mathfrak{X})$ of $\mathfrak{X}$. An association scheme $\mathfrak{X}$ is called *triply transitive* if $\mathrm{Aut}(\mathfrak{X})$ acts transitively on $V$ and $T_{0,v} = T_v = \widetilde{T}_v$ for $v \in V$.

This talk investigates the structure of the Terwilliger algebra of quasi-thin association schemes. In particular, we establish necessary and sufficient conditions for a quasi-thin association scheme to be triply transitive.

**ADEBOLA OMOTOYE**, University of Regina
*On the Inclusion of the Terwilliger Algebra in the Centralizer Algebra of an Association Scheme*

We present the tools for analyzing the extent to which the algebraic structure of the Terwilliger algebra $(T = T_x(\mathcal{S}))$ of an association scheme $(X, \mathcal{S})$ with respect to a vertex $x \in X$ can be obtained from that of the centralizer algebra $\tilde{T} = \tilde{T}_x(\mathcal{S})$ with respect to the same $x$ of the scheme. Since $T \subseteq \tilde{T}$ is known, it suffices to determine the decomposition of the restrictions of the irreducible $\tilde{T}$-modules to $T$. We demonstrate these calculations using schemes obtained from graphs, groups, and orbitals. In many cases, these restrictions remain irreducible, but as we will show, there are exceptions in the conjugacy class schemes of certain small groups.

**SAROBIDY RAZAFIMAHATRATRA**, University of Lethbridge
*The group association scheme of the Möbius group*

In this talk, I will present results on the group association scheme of the Möbius group over finite field. First, I will show that the automorphism group of the scheme is generated by the left-regular and the right-regular representations of the Möbius group, together with the anti-automorphism mapping an element to its inverse. Then, I will discuss the triple transitivity of the scheme.

**ALYSSA SANKEY**, University of New Brunswick
*An infinite family of linked strongly regular graphs realized through fusions of H(n,2)*

We describe a set of seven strongly regular graphs, linked by Seidel switching and decomposition. This group of seven (sorry) is part of an infinite family of such, examples of which are obtained by means of the symplectic and orthogonal graphs over GF(2). The same graphs arise through fusions of the binary Hamming scheme $H(n, 2)$ which were characterized by Muzichuk. The two of interest here are $F_1 : [0][1, 2, 5, 6, \ldots][3, 4, 7, 8, \ldots]$ and $F_2 : [0][1, 4, 5, 8, \ldots][2, 3, 6, 7, \ldots]$ where the notation presumes the standard distance-regular ordering of relations. We denote the two SRGs obtained in this way (along with their complements) $\Gamma^+$ and $\Gamma^-$. We find that $F_1$ produces $\Gamma^+$ when $n$ is congruent to 0 or 2 modulo 8, and $\Gamma^-$ when $n$ is congruent to 4 or 6. Similarly, $F_2$ produces $\Gamma^+$ or $\Gamma^-$ depending on $n$ modulo 8. We establish existence of certain cocliques in these graphs from which a Hamming scheme may be constructed, thereby providing an explicit fission of $\Gamma^\epsilon$ into $H(n, 2)$. Thus we have a combinatorial proof that these fusions exist for all even $n$, and a geometric understanding of the resulting graphs. Joint work with Caroline Vanicek.

**BLAKE SHIRMAN**, York University
*Instantaneous uniform mixing in tournament schemes*

Following the examples of Chan (2011) and Godsil, Mullin, and Roy (2017), we search for graphs with uniform mixing by first exploring the Bose-Mesner algebra of a 2-class association scheme. While these authors looked at *symmetric* 2-class schemes, which are constructed around strongly regular graphs, we focus on the surprisingly simpler *non-symmetric* 2-class schemes, which correspond to doubly regular tournaments. We start by defining the Hamiltonian of a graph, the continuous-time quantum walk, and how to identify uniform mixing in this walk. We naively investigate uniform mixing on the smallest doubly regular tournament: Cay$(\mathbb{Z}_3, \{1\})$. Immediately we are confronted with a technical problem: the conventional Hamiltonian can*not* achieve mixing on an oriented graph with odd order (e.g. *every* doubly regular tournament). However, we have no attachment to this convention, and instead use a "general Hamiltonian," showing that Cay$(\mathbb{Z}_3, \{1\})$ has uniform mixing for several infinite families of alternative Hamiltonians. We return to the methods of Chan, Godsil, et al. to fully characterize which non-symmetric 2-class association schemes have complex Hadamard matrices in their Bose-Mesner algebra *and* which of those schemes have uniform mixing *and* for what choices of Hamiltonian. We close with remarks about extending uniform mixing results with the general Hamiltonian, and future research goals.

**PAUL TERWILLIGER**, U. Wisconsin-Madison
*Circular Hessenberg pairs and the tridiagonal relations*

A square matrix is said to be Hessenberg whenever each entry below the subdiagonal is zero, and each entry on the subdiagonal is nonzero. A Hessenberg matrix is called circular whenever the top-right corner entry is nonzero, and every other entry above the superdiagonal is zero. A circular Hessenberg pair consists of two diagonalizable linear maps on a nonzero, finite-dimensional vector space, that each act on an eigenbasis of the other one in a circular Hessenberg fashion. In 2022, Jae-ho Lee conjectured that a circular Hessenberg pair satisfies two relations called the tridiagonal relations. In this talk, we prove Lee's conjecture. Our proof is not elementary. This is joint work with Kazumasa Nomura.

**LUC VINET**, Université de Montréal