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Uniform Quantitative Projection Theorems for Self Similar Sets

The classical Besicovitch–Federer projection theorem states that purely k-unrectifiable sets of finite k-dimensional Hausdorff measure have projections of zero measure in almost every direction. A quantitative problem is to determine how rapidly the measures of projected neighbourhoods decay, and if the estimates can hold uniformly.

In this talk, I will describe explicit self similar constructions exhibiting uniform logarithmic projection decay in arbitrary dimensions. For every $1 \leq k \leq d$ and $\alpha \in (0, 1)$, we construct a finite homogeneous iterated function system satisfying the open set condition. The resulting attractor has positive and finite k-dimensional Hausdorff measure, while the k-dimensional measures of the projections of its δ-neighbourhood decay at the rate $O((\log(1/\delta))^{-\alpha})$, uniformly over all k-dimensional subspaces as $\delta \to 0$.

We also explore the relationship between Favard length and analytic capacity in the planar setting. For the planar self similar sets underlying our constructions, we establish a sharp logarithmic capacity law and a quantitative Vitushkin inequality with exponent 2α. These comparisons are superlinear for $\alpha > 1/2$, with exponents arbitrarily close to two. This is a joint work with Layna Mangiapanello, Caleb Marshall, and Victoria Risner.