Algebraic combinatorics: longstanding questions in symmetric functions
Combinatoire algébrique: questions de longue date sur les fonctions symétriques
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MOHAMMAD BARDESTANI, John Abbott College
Chromatic Quasisymmetric Functions and Counting Matrices in Nilpotent Hessenberg Lie Algebras

To every Hessenberg function one can associate a nilpotent Hessenberg Lie algebra. Counting matrices with a prescribed Jordan form in these Lie algebras is closely connected to chromatic quasisymmetric functions. In this talk, we explain this connection and describe how combinatorial data arising from chromatic quasisymmetric functions encodes information about the geometry and representation theory of Hessenberg varieties. Moreover, a Hessenberg function naturally determines an element in the Kazhdan–Lusztig basis of the Hecke algebra of the symmetric group. For certain families of Hessenberg functions, we show how the functions counting matrices with prescribed Jordan form in the corresponding nilpotent Hessenberg Lie algebras can be used to study the values of irreducible characters on these Kazhdan–Lusztig basis elements.

HOUCINE BEN DALI, Harvard
About Goulden–Jackson's Matching-Jack and b-conjectures

The Matching-Jack conjecture and the b-conjecture were introduced by Goulden and Jackson in 1996. They suggest that certain series of Jack polynomials in three alphabets are generating functions of maps on general surfaces (orientable or not), counted with a nonorientability weight depending on the Jack parameter b. At $b = 0$ and $b = 1$ they reduce to classical representation theory results on orientable maps (Schur functions) and on general maps (zonal polynomials). For general b, both conjectures are still open.

I will present some partial results, and then discuss connections to other problems, including Lassalle's conjecture on Jack characters, b-Hurwitz numbers, and the Macdonald shuffle theorems.

FRANÇOIS BERGERON, Université du Québec à Montréal
A (q,t)-Overview of q-analogs

By a simple homogenization process, one may turn a q-analog into a symmetric (q, t)-analog. Although simple, this bijective correspondence makes many concepts and identities become more natural, and proofs follow readily from classical properties of symmetric functions in two variables. A more intricate non-homogeneous extension is also developed. We illustrate this approach, revisiting recent developments in the study of γ-positivity, Lucas analogues, and the monoid of Cyclotomic generating functions. This also leads naturally to new constructions and conjectures. We further explore other avenues of investigations, included graded and equivariant γ-positivity and γ-anti-positivity (also known as alternatingly γ-positive).

HONG CHEN, UCLA
Characterizing Macdonald's Hypergeometric Series

Macdonald introduced multivariate hypergeometric series associated with Jack and Macdonald polynomials, extending the classical hypergeometric series and their zonal polynomial analogues arising in multivariate statistics. In this talk, we establish characterizations of these series by differential and q-difference equations. In the Jack setting, our results generalize earlier work on hypergeometric functions of matrix argument and subsequent developments of Yan and Kaneko. We also present q-difference characterizations in the Macdonald setting. These results provide a common perspective on hypergeometric series through the operators associated with Jack and Macdonald polynomials.

PATTY COMMINS, LACIM, Université du Québec à Montréal
Principal specializations of the t-chromatic symmetric function

We introduce the t-chromatic symmetric function, a variant of Stanley's chromatic symmetric function in which the variable t records the number of distinct colors used in a proper coloring. Although this t-variant is equivalent to the classical chromatic symmetric function as a graph invariant, its principal specializations retain substantially more information.

Our construction is motivated by the work of Bajo–Beck–Vindas-Meléndez on the q-chromatic polynomial and the conjecture of Loehr–Warrington that the nth principal specialization of the chromatic symmetric function distinguishes trees on at most n vertices, both of which stem from Stanley's longstanding question of whether the classical chromatic symmetric function distinguishes trees. We prove that the third principal specialization of the t-chromatic symmetric function distinguishes caterpillars with three spine vertices as well as elongated bistars. We conjecture, based on computational data, that the fourth principal specialization distinguishes all caterpillars.

This is joint work with Aram Bingham, Kyle Celano, Rosa Orellana, Jianping Pan, and Mary Claire Simone.

LUCAS GAGNON, University of Southern California
Formulas for Koornwinder Polynomials

This talk will consider the problem of combinatorial formulas for nonsymmetric Koornwinder polynomials, which are the nonsymmetric Macdonald polynomials of the non-reduced type $(C_n^\vee, C)$. I'll start by presenting a tableau formula based on alcove walks; while this formula is ostensibly combinatorial it contains a huge number of terms. Then I will discuss the problem of compressing the tableau formula into something closer to the established fillings formula for classical (type A) Macdonald polynomials. Finally I will apply compression techniques, with varying levels of success, to Koornwinder polynomials and state the resulting 'compression formula.' Based on joint work with Laura Colmenarejo and Arun Ram.

MEGUMI HARADA, McMaster University
Dimensions of type A Hessenberg varieties over a fixed sheet

This talk is on joint work with Martha Precup and Colleen Robichaux.

Hessenberg varieties $\mathcal{H}\mathrm{ess}(\mathsf{X}, \mathbf{h})$ are subvarieties of the flag variety parameterized by a Hessenberg function $\mathbf{h} : [n] \to [n]$ and a matrix $\mathsf{X} \in \mathfrak{gl}_n(\mathbb{C})$. Hessenberg varieties are known to be connected to the theory of quasisymmetric functions through the study of their cohomology rings. In recent work, Goldin and Precup showed the existence of flat degenerations of Hessenberg varieties to nilpotent Hessenberg varieties over the minimal sheet. This implies that all Hessenberg varieties over the minimal sheet have the same dimension. We generalize this dimension result to arbitrary sheets. Specifically, we prove that for a fixed Hessenberg function $\mathbf{h} : [n] \to [n]$, all Hessenberg varieties $\mathcal{H}\mathrm{ess}(\mathsf{X}, \mathbf{h})$ defined in the type A flag variety by linear operators X from the same sheet of the Lie algebra $\mathfrak{gl}_n(\mathbb{C})$ have the same dimension.

JOHN LENTFER, University of California, San Diego
Sign components for general bosonic-fermionic coinvariant rings

We prove the sign-isotypic components of the coinvariant rings $R_n^{(k,j)} \otimes R_n^{(0,1)}$ and $R_n^{(k,j+1)}$ are isomorphic, establishing the hook-components conjecture of F. Bergeron (2024). In the important case of $R_n^{(2,0)} \otimes R_n^{(0,1)}$ and $R_n^{(2,1)}$, this establishes the sign-character component of a conjecture of Zabrocki (2019) on a module for the Delta conjecture, and computes the graded dimensions of the Khovanov–Rozansky homology of the $(n, n+1)$-torus knot. This is based on joint works with Nicolle González, Hanna Mularczyk, and Pablo S. Ocal.

JINTING LIANG, University of British Columbia
Symmetric quasisymmetric Schur-like functions

The area of quasisymmetric Schur-like functions had its genesis in quasisymmetric Schur functions, which naturally arose in the study of Macdonald polynomials and refined both Schur functions and their properties. One natural question is when are these quasisymmetric Schur-like functions symmetric. In this talk we classify when (row-strict) dual immaculate functions and (row-strict) extended Schur functions, as well as their skew generalizations, are symmetric. We also classify when their natural variants, termed advanced functions, are symmetric. In every case our classification recovers classical skew Schur functions. This is joint work with Maria Esipova and Stephanie van Willigenburg.

JIKE LIU, Rutgers - New Brunswick
Four-Functions Inequalities, Skeps, and Schur Positivity

In 2007, Lam, Postnikov and Pylyavskyy proved that $s_\lambda s_\mu \leq_s s_{\lambda\vee\mu}\, s_{\lambda\wedge\mu}$, where $f \leq_s g$ means that $g-f$ is Schur positive, and conjectured more generally that $s_\lambda s_\mu \leq_s s_{\lambda'} s_{\mu'}$ whenever $\lambda'+\mu' = \lambda+\mu$ and the pair (λ',μ') is more balanced than (λ,μ). Speyer recently proved this conjecture using skeps, a new combinatorial model for Littlewood–Richardson coefficients. In another direction, Chan, Chen, Pak and Soskin proved a Schur-positive analogue of the Ahlswede–Daykin four-functions theorem, and conjectured a Schur-positive analogue of a midpoint inequality of Klartag and Lehec. We prove a single Schur-positive four-functions theorem that contains all of these: if nonnegative weights a, b, c, d on partitions satisfy $a(\lambda)\, b(\mu) \leq c(\lambda')\, d(\mu')$ whenever (λ',μ') is the image of (λ,μ) under a fixed certain pairing, then $\big(\sum_\lambda a(\lambda)s_\lambda\big)\big(\sum_\lambda b(\lambda)s_\lambda\big) \leq_s \big(\sum_\lambda c(\lambda)s_\lambda\big)\big(\sum_\lambda d(\lambda)s_\lambda\big)$. Suitable choices of the pairing and the weights give the Chan–Chen–Pak–Soskin inequality, Speyer's theorem, and the conjectured Schur-positive Klartag–Lehec inequality. This also applies to sign-normalized stable Grothendieck polynomials and to dual stable Grothendieck polynomials, giving four-functions inequalities in the Schur order for both families. This is joint work with Swee Hong Chan.

ROSA ORELLANA, Dartmouth College
Proper implies improper for the chromatic symmetric function

Stanley's question of whether the chromatic symmetric function distinguishes trees has been unanswered for over thirty years. It has now been checked for trees with at most 29 vertices, and for several classes of trees. There is the perception that answering the question might be easier for the class of "proper" trees, i.e. trees where every vertex is either a leaf or adjacent to a leaf. In this abstract, we show that an affirmative answer for a subclass of proper trees, which we call "k-weighted", is equivalent to an affirmative answer for all trees. In addition, we show that in the class of k-weighted trees, the coefficients carry redundant information.

STEPHAN PFANNERER, University of Waterloo
Skew Schur functions and a mystery cyclic action

Skew Schur functions encode skew characters of the symmetric group through their power-sum expansions. For rectangular shapes, Brendon Rhoades famously showed that, after a suitable sign adjustment, these character values count fixed points of powers of promotion on standard Young tableaux, giving an instance of the cyclic sieving phenomenon. Consequently, the sign-adjusted character associated with a rectangular shape, restricted to the cyclic subgroup generated by a long cycle, is the character of a permutation action.

We thus ask: when does the restriction of a skew character to this cyclic subgroup arise from a permutation action? We show that this happens precisely when all values of the restricted character are nonnegative.

Our result applies to arbitrary skew shapes, but while Rhoades provides the group action through promotion for rectangular shapes, finding a natural cyclic action on standard skew tableaux remains an open question.

Based on joint work with Per Alexandersson, Martin Rubey, and Joakim Uhlin.

JERÓNIMO VALENCIA PORRAS, University of Waterloo
Type C multiline queues and the open-boundary TASEP

The totally asymmetric simple exclusion process (TASEP) is a finite Markov chain of particles hopping between adjacent sites on a one-dimensional lattice. The multispecies TASEP is a generalization in which particles have different types. These processes have interesting connections to algebraic combinatorics: the stationary distribution of the TASEP on a circle is connected to Macdonald polynomials at t=0, whereas the stationary distribution of the open-boundary TASEP is connected to Koornwinder polynomials at t=0. Multiline queues were introduced by Ferrari and Martin (2007) to compute the stationary distribution of the multispecies TASEP on a circle. It has been a long-standing open problem to find a combinatorial formula for the stationary distribution of the multispecies TASEP with open boundaries. Recently, we studied the combinatorics of Ferrari—Martin multiline queues using type A crystals. In this talk, we use crystals of type C to construct an analog of multiline queues and give a combinatorial formula for the stationary distribution of the multispecies open-boundary TASEP for a certain specialization of the boundary parameters. This is joint work with Olya Mandelshtam.

SOPHIE REHBERG, Univerité du Québec à Montréal
Chromatic word-quasisymmetric functions of matroids

Billera, Jia, and Reiner (2009) introduced the quasisymmetric functions of matroids and showed that this defines a Hopf alegebra homomorphism which is a valuative invariant, i.e., isomorphic matroids have the same quasisymmetric function and polytopal decompositions of matroid base polytopes define relations among the corresponding quasisymmetric functions. In this project we study an analogue in non-commuting variables, the word-quasisymmetric functions. To every matroid M we associate a word-quasisymmetric function $\psi(M)$ and call this the chromatic word-quasisymmetric functions of a matroid. Matroids and word-quasisymmetric functions form Hopf algebras, and our map ψ between them is a homomorphism. We want to study the kernel, equivalently the image, of the map ψ from matroids to word-quasisymmetric functions, that is, we would like to understand which matroids are indistinguishable by the chromatic word-quasisymmetric functions. The map ψ is not an invariant, but we can show that it is valuative. Using Schubert matroids and nested matroids, special classes of matroids, we prove a lower bound of $2^d - d$ for the rank of the map ψ from matroids to the chromatic word-quasisymmetric functions in degree d and conjectured the upper bound of $d!$ is tight. Recent AI supported work shows that the conjecture is true. This is joint work with Raul Penaguiao.

MARINO ROMERO, Minnesota State University, Mankato
Wreath Macdonald Polynomials - Operators and Identities

Wreath Macdonald polynomials are multisymmetric functions generalizing modified Macdonald polynomials. Whereas modified Macdonald polynomials correspond to representations of the symmetric group S_n and torus fixed points on the Hilbert scheme of points on the plane, the wreath version corresponds to $S_n \wr \mathbb{Z}/r\mathbb{Z}$ representations and $\mathbb{Z}/r\mathbb{Z}$ fixed loci in the Hilbert scheme. The existence, integrality, and positivity of wreath Macdonald polynomials was conjectured by Haiman in 2002. In 2014, these conjectures were proved by Bezrukavnikov and Finkelberg using algebraic geometric methods.

This talk is based on joint work with Josh J. Wen, where we develop computational tools for dealing with wreath Macdonald polynomials. In particular, we will focus on describing their Pieri rules and describe how we prove this by giving wreath analogues of Tesler's Identity, nabla and Delta operators, and Garsia-Mellit's Five Term Relation.

KARTIK SINGH, University of Waterloo
Multiline queue crystals, FM projection, and RSK

Multiline queues were introduced by Ferrari and Martin to compute the stationary probabilities of the totally asymmetric simple exclusion process via fibers of the type map. Viewed as binary matrices, they carry two commuting crystal structures, with dual-RSK realized by the collapsing map. We characterize how the type changes under the crystal operators on the multiline queue crystal, allowing us to identify the type map on nonwrapping multiline queues with the right-key map under the natural crystal isomorphism with semistandard Young tableaux. We further show that the type of a multiline queue is determined by the type of its collapsed image together with a recording tableau. As applications, we prove Demazure-atom positivity at $t = 0$ for ASEP polynomials and permuted-basement Macdonald polynomials, as well as quasisymmetric-Schur positivity for quasisymmetric Macdonald polynomials.

JOSHUA SWANSON, IDA Center for Communications Research, La Jolla
On the recent proof of the Shareshian–Wachs conjecture

On October 6th, 2026, OpenAI released solutions to hundreds of problems in pure mathematics. Manuscript number 169 proved the well-known Shareshian–Wachs Conjecture. This problem asked for a q-analogue of the Stanley–Stembridge Conjecture (now Hikita's Theorem) regarding e-positivity of the chromatic symmetric function associated to a natural unit interval graph. We will give a high-level overview of the proof, which connects the problem to the Θ-basis of Gross–Hacking–Keel–Kontsevich in certain scattering diagram cluster algebras, together with a q-analogue of that theory due to Davison–Mandel. We will also discuss the ongoing community response to these developments.

VASU TEWARI, University of Toronto
Set-valued LLT polynomials and the valley delta conjecture

I will present a proof of the compositional valley Delta conjecture by using the set-valued LLT polynomials introduced by Andy Wilson in unpublished work. The techniques are elementary and classical, i.e. we do not need the Carlsson-Mellit algebra. This is joint work with Andy Wilson.

MICHELLE WACHS, Miami

DORA WOODRUFF, MIT
Honeycombs and Sums of Hermitian Matrices, Revisited

The Horn Problem is a classic problem, dating back to at least 1912. If A and B are two Hermitian matrices with known eigenvalue spectra, it asks about the possible spectra of $A+B$. The problem was fully resolved by Knutson, Tao, and Klyachko using tools from Schubert calculus and representation theory. An important component of the solution was honeycombs, a combinatorial model for Littlewood-Richardson coefficients developed by Knutson and Tao. Their key theorem states that a honeycomb with given boundary values exists if and only if there exists a triple $(A, B, A + B)$ of Hermitian matrices with corresponding eigenvalue spectra. This theorem gives a pleasant combinatorial and geometric answer to the original Horn problem. We present a new proof of this bridge between honeycombs and Hermitian matrices. The main surprise is that the proof can be made very elementary. It bypasses Schubert calculus and representation theory, making use only of linear algebra, basic calculus, and combinatorics. Our proof also suggests new perspectives on Littlewood-Richardson coefficients. This project is joint work with Ankur Moitra and Alex Postnikov.